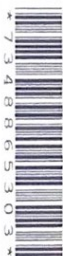


GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Thursday 25 May 2017 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper



First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Calculate.

(a) $\sqrt{\frac{4.8^2 + 3.6^2}{4}}$

(a) 3 [2]

(b) $\frac{1}{(2 \times 10^4) + (5 \times 10^3)}$

(b) 4×10^{-5} [2]

2 The length, L , of a steel rod is 8.3 m, correct to 1 decimal place.

Complete the error interval for length L .

..... 8.25 $\leq L <$ 8.35 [2]

3

- 3 (a) Write 504 as the product of its prime factors.

$$504 \div 2^3 = 63$$

$$63 = 7 \times 9$$

$$9 = 3^2$$

$$504 = 2^3 \times 3^2 \times 7$$

(a) $2^3 \times 3^2 \times 7$ [3]

- (b) Find the lowest common multiple (LCM) of 180 and 504.

$$180 = 2^2 \times 3^2 \times 5$$

$$504 = 2^3 \times 3^2 \times 7$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

(b) 2520 [2]

4

- 4 Find the value of s when $u = 12$, $a = 10$ and $t = 4$.

$$s = ut + \frac{1}{2}at^2$$

$$S = (12 \times 4) + \left(\frac{1}{2} \times 10 \times 4^2\right) = 128$$

128

..... [2]

- 5 Mo's tyre pressure gauge shows a reading which is 12% higher than the actual pressure.

What is the actual pressure when Mo's gauge shows 38.64?

$$38.64 \div 1.12 = 34.5$$

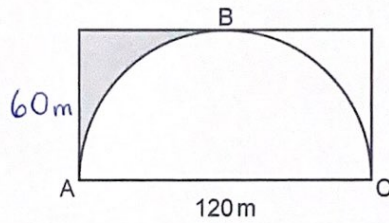
34.5

..... [3]

5

- 6 The diagram shows a semi-circle inside a rectangle of length 120m. The semi-circle touches the rectangle at A, B and C.

Not to scale



Calculate the **perimeter** of the shaded region.
Give your answer correct to 3 significant figures.

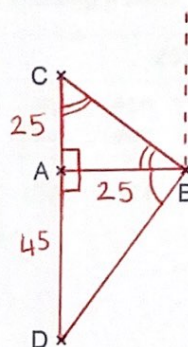
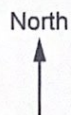
$$\begin{aligned} \text{Circumference of a circle} &= \pi d = 120\pi \\ \text{Circumference of } \frac{1}{4} \text{ of the circle} &= \frac{1}{4} \times 120\pi \\ &= 30\pi \end{aligned}$$

$$\begin{aligned} 60 + 60 + 30\pi &= 214.25 \\ &= 214 \text{ 3 s.f.} \end{aligned}$$

..... 214 m [5]

- 7 A, B, C and D are four towns.

B is 25 kilometres due East of A.
 C is 25 kilometres due North of A.
 D is 45 kilometres due South of A.



Not to scale

- (a) Work out the bearing of B from C.

$$\frac{180 - 90}{2} = 45$$

$$\text{B from C} = 90^\circ + 45^\circ = 135^\circ$$

(a) 135 ° [2]

- (b) Calculate the bearing of D from B.

$$\tan(\angle ABD) = \frac{45}{25}$$

$$\angle ABD = \tan^{-1}\left(\frac{45}{25}\right) = 60.95$$

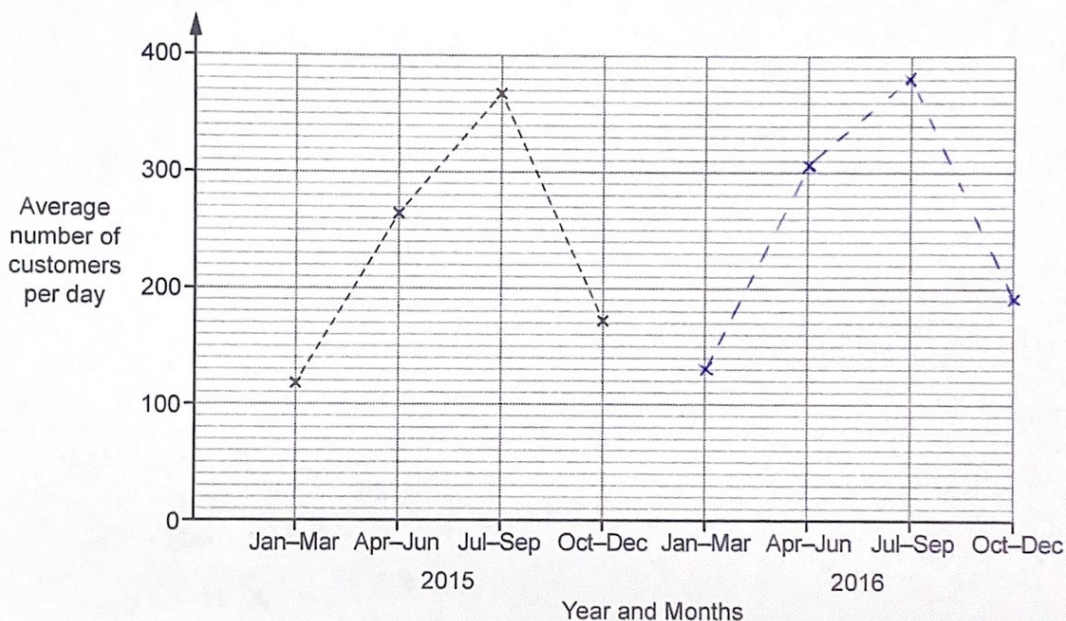
$$360 - 90 - 60.95 = 209.1^\circ$$

(b) 209.1 ° [4]

- 8 The table shows the average number of customers per day entering a shop.

	2015				2016			
Months	Jan-Mar	Apr-Jun	July-Sep	Oct-Dec	Jan-Mar	Apr-Jun	July-Sep	Oct-Dec
Average number of customers per day	119	264	368	172	130	304	381	192

- (a) Complete the time series graph below.



[2]

- (b) Make two different comments comparing the number of customers entering the shop in 2015 and 2016.

Comment 1 There were more people shopping in
2016.

Comment 2 There were more customers in the summer
months in 2016.

[2]

- 9 Each week Dan drives two routes, route X and route Y.

One week he drives route X three times and route Y twice.
He drives a total of 134 miles that week.

Another week he drives route X twice and route Y five times.
He drives a total of 203 miles that week.

- (a) Find the length of each route.

$$\begin{aligned} 3X + 2Y &= 134 \\ 2X + 5Y &= 203 \end{aligned}$$

$$Y = \frac{134 - 3X}{2}$$

sub Y into $2X + 5Y = 203$

$$\Rightarrow 2X + \frac{5(134 - 3X)}{2} = 203$$

$$\Rightarrow 2X + \frac{670 - 15X}{2} = 203$$

$$\Rightarrow \frac{4X}{4X} + \frac{670 - 15X}{2} = 203$$

$$\Rightarrow -11X = -264$$

$$\Rightarrow X = 24$$

$$\Rightarrow Y = 31$$

(a) route X = $\frac{24}{\dots\dots\dots}$ miles

route Y = $\frac{31}{\dots\dots\dots}$ miles [5]

- (b) State an assumption that has been made in answering part (a).

The distance for each route is the same
every day. [1]

- 10 On 1st November 2015 there were 4200 trees planted in a wood.
On 1st November 2016, only 3948 of these trees were still alive.

It is assumed that the number of trees still alive is given by

$$N = ar^t$$

where N is the number of trees still alive t years after 1st November 2015.

- (a) Write down the value of a .

(a) 4200 [1]

- (b) Show that r is 0.94. [2]

$$3948 = 4200 r^1$$

$$r = \frac{3948}{4200} = 0.94$$

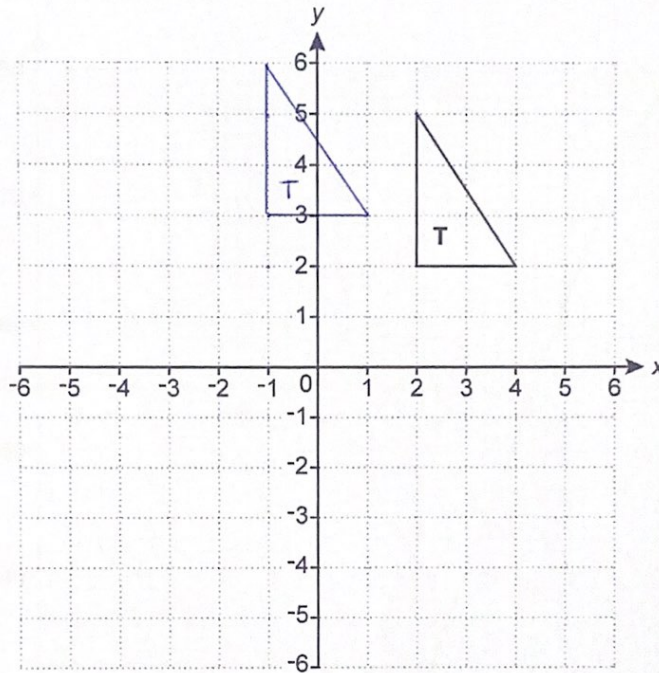
- (c) Show that on 1st November 2030 the number of trees still alive is predicted to have decreased by over 60% compared with 1st November 2015. [3]

$$\begin{aligned} \text{Trees on 1/11/2030} &= 4200 \times r^{15} \\ &= 1660 \end{aligned}$$

$$\frac{1660}{4200} \times 100 = 39.5\%$$

Hence the number of trees will have decrease by 60.5%.

- 11 Triangle T is drawn on a coordinate grid.



- (a) Translate triangle T using the vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$. [2]
- (b) Describe fully the **single** transformation that represents the following.
- (i) A rotation with centre (0, 0) of 180° followed by a rotation with centre (0, 0) of 90° clockwise.

A rotation with centre (0,0) of 90°

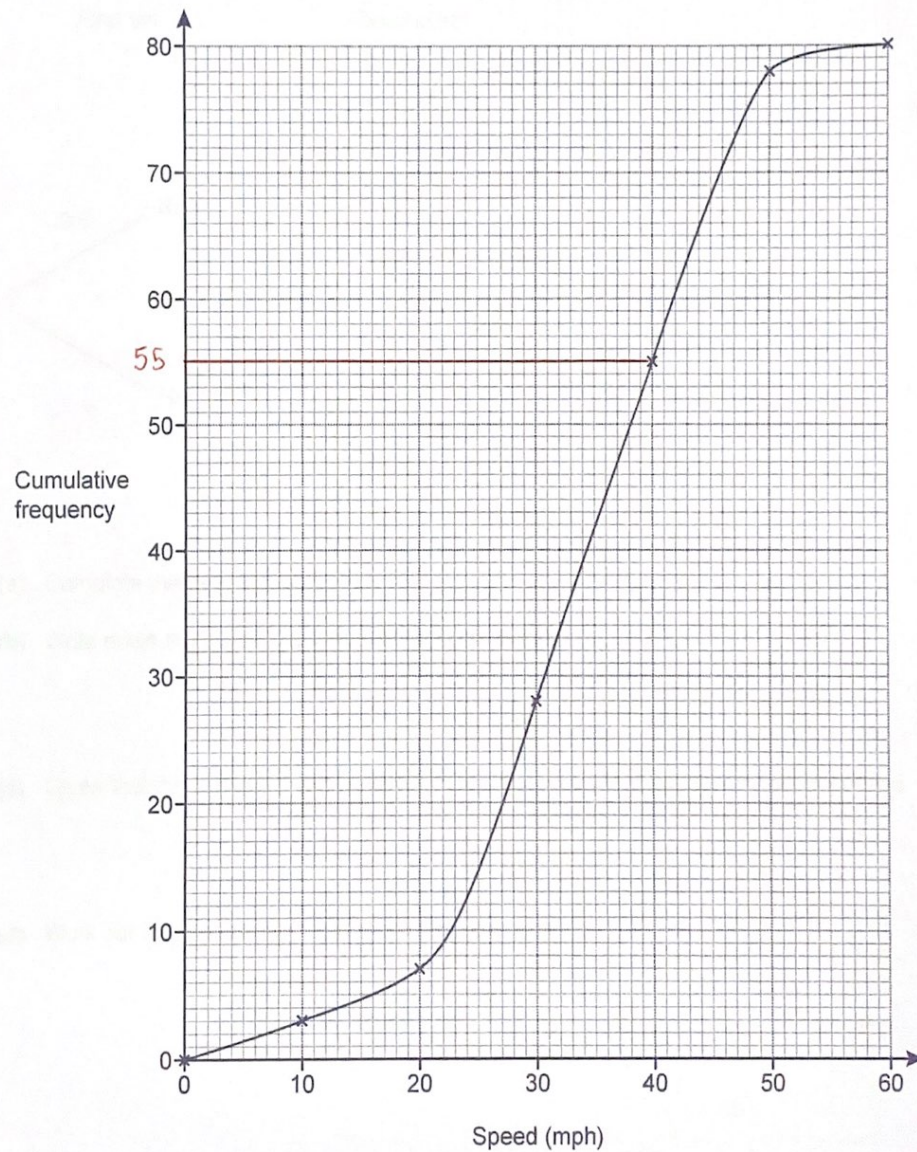
 anti clock wise. [2]

- (ii) A reflection in the x-axis followed by a reflection in the y-axis.

Rotation with centre (0,0) of 180°

 [3]

- 12 The cumulative frequency graph shows the speeds, in miles per hour (mph), of vehicles passing a 40 mph speed limit sign on a road.



A speed camera will be installed if more than 30% of vehicles go over the speed limit of 40 mph.

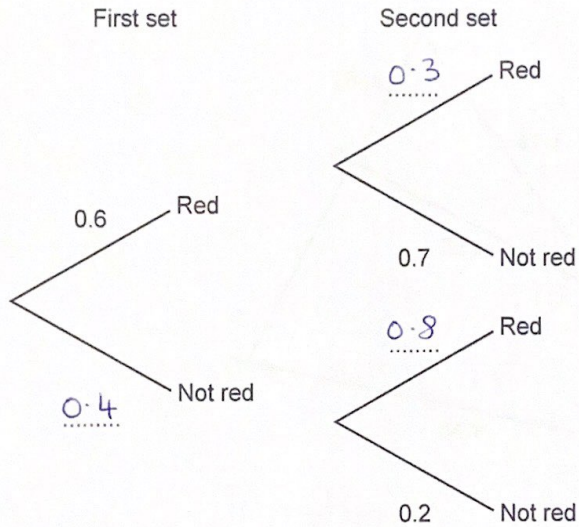
Use information from the graph to decide if a speed camera should be installed.

[4]

$$\frac{55}{80} = 68.75\% \text{ go } 40 \text{ mph or less}$$

so 31.25% go over, so a speed camera should be installed.

- 13 Rashid drives his car along a road passing through two sets of traffic lights. The tree diagram shows the probabilities of the lights being **red** when he reaches them.



- (a) Complete the tree diagram. [1]

- (b) Write down the probability that the first set is **not red**.

(b) 0.4 [1]

- (c) Given that the first set is **red**, write down the probability that the second set is **not red**.

(c) 0.7 [1]

- (d) Work out the probability that both sets are **not red**.

$$0.4 \times 0.2 = 0.08$$

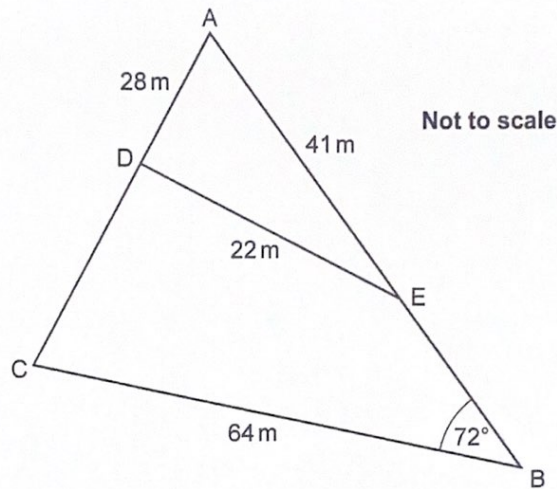
(d) 0.08 [2]

- (e) Work out the probability that at least one set is **not red**.

$$(0.6 \times 0.7) + (0.4 \times 0.8) + (0.4 \times 0.2) = 0.82$$

(e) 0.82 [3]

- 14 The diagram shows triangle ABC with D on AC and E on AB. DE is a straight line.



AD = 28m, AE = 41m, DE = 22m and BC = 64m.

Calculate the length CD.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos DAE = \frac{28^2 + 41^2 - 22^2}{2 \times 28 \times 41}$$

$$\Rightarrow DAE = 30.37$$

$$ACB = 180 - 72 - 30.37 = 77.63$$

$$\frac{\sin 72}{CA} = \frac{\sin 30.37}{64}$$

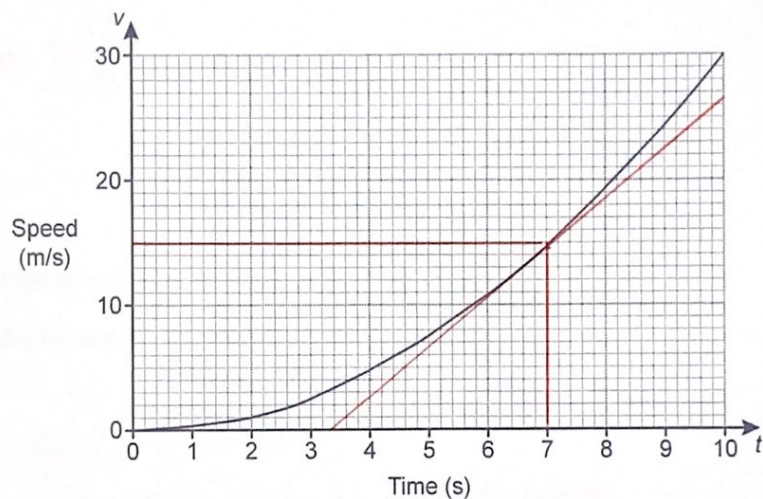
$$\Rightarrow CA = \frac{\sin 72}{\frac{\sin 30.37}{64}} = 120.39$$

$$CD = 120.39 - 28 = 92.39$$

92.39

..... m [6]

- 15 The graph shows the speed, v metres per second (m/s), of a car at time t seconds.



- (a) Find the speed of the car at $t = 7$.

(a)15..... m/s [1]

- (b) It is claimed that the car has accelerated from 0 to 60 miles per hour in the first 10 seconds.

Does the graph support this claim? Show your reasoning.
Use 1 mile = 1.6 kilometres.

[5]

$$\begin{aligned}
 30 \text{ m/s} &= 0.03 \text{ km/s} \\
 &= 108 \text{ km/h} \\
 &= \left(\frac{108}{1.6}\right) \text{ mph} \\
 &= 67.5 \text{ mph}
 \end{aligned}$$

So yes the graph does support the claim.

- (c) Use the graph to estimate the acceleration at $t = 7$.

$$\text{gradient of tangent} = \frac{15-0}{7-3.4} = 4.17$$

(c) 4.17 m/s² [3]

- (d) The speed of this car is directly proportional to the square of the time.

Find a formula linking v and t .

$$v = kt^2$$

where k is a positive integer

(d) $v = kt^2$ [3]

- (e) Georgina says that the graph shows that the speed of the car will continue to increase after 10 seconds.

Make one comment to show that this statement is incorrect.

..... You can only see up to 10 seconds on
 the graph and the car will eventually
 reach its maximum speed. [1]

16 Write $x^2 - 10x + 16$ in the form $(x+a)^2 + b$.

$$x^2 - 10x + 16 = (x - 5)^2 - 25 + 16 = (x - 5)^2 - 9$$

$$\dots\dots\dots (x-5)^2 - 9 \dots\dots\dots [3]$$

17 Describe fully the graph which has the equation $x^2 + y^2 = 9$.

$\dots\dots\dots$ Circle with centre (0,0) and radius 3 $\dots\dots\dots [2]$

18 (a) Solve by factorisation.

$$2x^2 + 5x - 12 = 0$$

$$(2x - 3)(x + 4) = 0$$

$$x = \frac{3}{2} \text{ and } x = -4$$

(a) $x = \frac{3}{2}$ or $x = -4$ [3]

(b) Solve this equation.
Give each value correct to 2 decimal places.

$$3x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times -3}}{2 \times 3}$$

$$= 0.72 \text{ and } -1.39$$

(b) $x = 0.72$ or $x = -1.39$ [3]

- 19 (a) Here are the first four terms of a sequence.

$$\frac{1}{2} \quad \frac{4}{3} \quad \frac{9}{4} \quad \frac{16}{5}$$

Find the n th term of this sequence.

$$\begin{array}{cccc} 1 & 4 & 9 & 16 & = & n^2 \\ 2 & 3 & 4 & 5 & = & n+1 \end{array}$$

$$\frac{n^2}{n+1}$$

(a) [2]

- (b) Here are the first four terms of a quadratic sequence, the n th term of this quadratic sequence is $an^2 + bn + c$.

$$2 \quad 12 \quad 28 \quad 50$$

Find the values of a , b and c .

$$\begin{array}{cccc} 2 & 12 & 28 & 50 \\ \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ +10 & +16 & +22 & \\ \underbrace{\quad} & \underbrace{\quad} & & \\ +6 & +6 & & \end{array}$$

$$0^{\text{th}} \text{ term} = -2$$

$$a = 6 \div 2 = 3$$

$$c = 0^{\text{th}} \text{ term} = -2$$

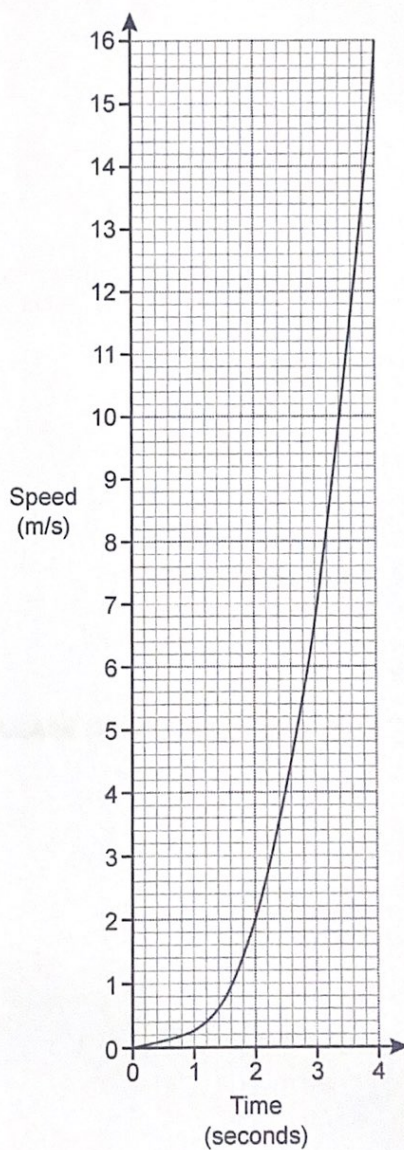
To find b :

$$\begin{aligned} (n=1): 3n^2 + bn + c &= 2 \\ \Rightarrow 3 + b + -2 &= 2 \\ \Rightarrow b &= 1 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad a &= \frac{3}{\dots\dots\dots} \\ b &= \frac{1}{\dots\dots\dots} \\ c &= \frac{-2}{\dots\dots\dots} \end{aligned}$$

[4]

- 20 The graph shows the speed, in metres per second, of a particle over the first four seconds of motion.



Use the graph to estimate the distance travelled by the particle in the four seconds.

distance travelled = area under graph.

18

..... metres [2]

END OF QUESTION PAPER