

Proof (Higher) Mark Scheme

1(a)	$4(2x - 3) - 2(2x + 9) = 8x - 12 - 4x - 18$	[1] Expanding brackets
	$= 8x - 4x - 12 - 18$ $\equiv 4x - 30$	[1] Grouping similar terms
1(b)	$(n - 1)^2 - (n - 2)^2$ $(n^2 - 2n + 1) - (n^2 - 4n + 4)$	[1] Expanding brackets
	$= n^2 - n^2 - 2n + 4n + 1 - 4$ $= 2n - 3$	[1] Grouping similar terms
1(c)	$(n + 2)(n + 2) - 3(n + 4)$ $= n^2 + 2n + 2n + 4 - 3n - 12$	[1] Expanding brackets
	$= n^2 + n - 12 + 4$ $\equiv (n + 4)(n - 3) + 4$	[1] Grouping similar terms
1(d)	$= 3(n^2 + 2n - 3) - 3 + 3n$	[1] Expanding brackets
	$= 3n^2 + 9n - 12$ $\equiv (3n - 3)(n + 4)$	[1] Grouping and cancelling similar terms
2(a)	$= 3n^2 + 9n + n + 3 - 3n^2 - 7n$	[1] Expanding brackets
	$= 3n + 3$ $= 3(n + 1)$	[1] Grouping and cancelling similar terms
2(b)	$n^2 + 6n + 9 - 3n - 4$	[1] Expanding brackets
	$= n^2 + 3n + 5$ $(n + 1)(n + 2) + 3$	[1] Grouping and cancelling similar terms
2(c)	$= n^2 - 6n + 9 - 2n - 1$	[1] Expanding brackets
	$= n^2 - 8n + 8$ $= (n - 4)^2 - 8$	[1] Grouping and cancelling similar terms
3(a)	$2n \times 2m = 4nm$	[1] Let n and m be any integers so that 2n and 2m are both even numbers.
	$= 2(2nm)$ which is even	[1] Answer as required
3(b)	$(2n + 1)(2m + 1) = 4nm + 2n + 2m + 1$	[1] Let n and m be any integers so that 2n+1 and 2m+1 are both odd numbers.
	$= 2(2nm + n + m) + 1$ which is odd	[1] Answer as required
3(c)	$(2n + 1)(2n + 3)(2n + 5)$ $= (4n^2 + 2n + 6n + 3)(2n + 5)$	[1] Creation of correct algebraic expression
	$(4n^2 + 2n + 6n + 3)(2n + 5)$ $= 8n^3 + 36n^2 + 46n + 15$	[1] Expanding brackets
	$= 2(4n^3 + 18n^2 + 23n + 7) + 1$ simplifies to $2(n) + 1$	[1] Factorising to show its always odd

Turn over ►

4(a)	$(2n + 1) + (2m + 1) + (2p + 1)$	[1] Creation of correct algebraic expression
	$= 2(n + m + p + 1) + 1$ $= 2(x) + 1$	[1] Factorising to show its always odd
4(b)	$(2n + 1)^2 + (2m + 1)^2 = 4n^2 + 4n + 1 + 4m^2 + 4m + 1$	[1] Creation of correct algebraic expression and expanding brackets
	$2(2n^2 + 2m^2 + 2n + 2m + 1)$	[1] Factorising to show its always odd
4(c)	$(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$	[1] Creation of correct algebraic expression and expanding brackets
	$= 2n + 1$	[1] Simplifying to final answer
5(a)	$= n^2 + 3n + 3n + 9 + 3n - n^2 - 3n - 12$	[1] Expanding brackets
	$= 6n - 3$ $= 3(2n - 1)$	[1] Grouping similar terms
5(b)	If a number is n , then the next number is $n + 1$ The sum is therefore $n + n + 1 = 2n + 1$	[1] Demonstration of logic
	By definition , $2n$ is even, and so $2n + 1$ must be odd.	[1] Final explanation
5(c)	$(5n)^2 + (5n + 5)^2 = 25n^2 + 25n^2 + 25n + 25n + 25$	[1] Expanding brackets
	$= 50n^2 + 50n + 25$	[1] Grouping similar terms
6(a)	$2n + (2n + 2) + (2n + 4)$	[1] Expanding brackets
	$= 6n + 6 = 6(n + 1)$	[1] Grouping similar terms and factorisation to show it is divisible by 6
6(b)	$= 16n^2 + 16n + 4 - 4n^2 - 8n - 4$	[1] Expanding brackets
	$= 12n^2 + 8n = 4(3n^2 + 2n)$	[1] Grouping similar terms and factorisation to show multiple of 4
6(c)	$= 4n^2 + 12n + 9 - 4n^2 + 12n - 9$	[1] Expanding brackets
	$= 24n = 8(3n)$	[1] Grouping similar terms and factorisation to show multiple of 8
7(a)	$(2n)^2 + (2n + 2)^2$	[1] Creation of correct algebraic expression
	$= 4n^2 + 4n^2 + 8n + 4$	[1] Expanding brackets
	$= 4(2n^2 + 2n + 1)$	[1] Grouping similar terms and factorisation to show multiple of 4

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7(b)	$(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$	[1] Creation of correct algebraic expression																									
	$= 2n + 1$	[1] Accept same proof with $2n - 1$																									
8(a)	$7x - (2x + 3)(x + 2) = 7x - (2x^2 + 4x + 3x + 6)$ $= -2x^2 - 6 = -(2x^2 + 6)$	[1] Expanding brackets and grouping similar terms																									
	$= -(2x^2 + 6)$	[1] Factorisation																									
	$-(2x^2 + 6)$ is always positive, so multiplying by a negative means that the answer is always negative, so Tom is correct.	[1] Suitable explanation of logic																									
8(b)	Changing the +3 into a -3 or +2 into a -2 would give a negative number when expanding the brackets but would then be made positive when multiplying by -1.	[1]																									
9(a)	$= (7 \times 2)^{20} - (7 \times 3)^2 = 7^{20} \times 2^2 - 7^2 \times 3^2$	[1] Changing of powers																									
	$= 7(7^{19} \times 2^2 - 7 \times 3^2)$	[1] Factorisation with 7 taken out																									
	A factor of 7 can be taken out, so the answer must be divisible by 7, and therefore a multiple of 7.	[1] Suitable explanation of logic																									
9(b)	3^{60} is always odd, because you are multiplying odd numbers. Also 25 is odd. So the difference between two odd numbers, i.e. subtracting them, is always even.	[1] Suitable explanation of logic																									
	$3^{60} - 25$ is going to be even. All even numbers are divisible by 2, so $3^{60} - 25$ is not prime.	[1] Suitable explanation of logic																									
10(a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td></tr> </tbody> </table> <p>Consider the new square shown,</p> $(44 - 33) \times (43 - 34) = 11 \times 9 = 99$	1	2	3	4	5	11	12	13	14	15	21	22	23	24	25	31	32	33	34	35	41	42	43	44	45	[1] Any example shown
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11	12	13	14	15																							
21	22	23	24	25																							
31	32	33	34	35																							
41	42	43	44	45																							
10(b)	<p>Taking the top left number as n, the other numbers can be written in terms of n.</p> $(n + 11 - n) \times (n + 10 - (n + 1))$ $= 11 \times (n + 10 - n - 1) = 11 \times 9$ $= 99$	[1] [1] [1] <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr><td>n</td><td>$n+1$</td></tr> <tr><td>$n+10$</td><td>$n+11$</td></tr> </tbody> </table>	n	$n+1$	$n+10$	$n+11$																					
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