MME

GCSE MATHEMATICS

AQA | Edexcel | OCR | WJEC

Proofs (Higher)

Please write clearly in block capitals

Forename:	
Surname:	

Materials

For this paper you must have:
- mathematical instruments

You ***can*** use a calculator.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- You may ask for graph paper, tracing paper and more answer paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

1 Show that the following statements are true:

1(a) $$4(2x - 3) - 2(2x + 9) \equiv 4x - 30$$

[2 marks]

Answer

1(b) $$(n - 1)^2 - (n - 2)^2 \equiv 2n - 3$$

[2 marks]

Answer

1(c) $$(n + 2)^2 - 3(n + 4) \equiv (n + 4)(n - 3) + 4$$

[2 marks]

Answer

1(d) $$3(n + 3)(n - 1) - 3(1 - n) \equiv (3n - 3)(n + 4)$$

[2 marks]

Answer

Turn over for next question

8

Turn over ►

2 Show that the following statements are true,

2(a)

$$(3n + 1)(n + 3) - n(3n + 7) \equiv 3(n + 1)$$

[2 marks]

Answer

2(b)

$$(n + 3)^2 - (3n + 4) \equiv (n + 1)(n + 2) + 3$$

[2 marks]

Answer

2(c)

$$(n - 3)^2 - (2n + 1) \equiv (n - 4)^2 - 8$$

[2 marks]

Answer

6

3(a) Prove the product of two even numbers is always even.

[2 marks]

Answer ______________________

3(b) Prove that the product of two odd numbers is always odd.

[2 marks]

Answer ______________________

3(c) Prove the product of three consecutive odd numbers is odd.

[3 marks]

Answer ______________________

7

Turn over ►

4(a) Prove algebraically that the sum of any three odd numbers is odd.

[2 marks]

Answer

4(b) Prove algebraically that the sum of the squares of two odd integers is always even.

[2 marks]

Answer

4(c) Prove that when two consecutive integers are squared, that the difference is equal to the sum of the two consecutive integers.

[2 marks]

Answer

Turn over for next question

6

5(a) Prove that,

$$(n+3)^2 + n(3-n) - 3(n+4)$$

is a multiple of 3 for all integer values of n.

[2 marks]

Answer

5(b) Prove algebraically that the sum of two consecutive numbers is odd.

[2 marks]

Answer

5(c) Prove algebraically that the sum of the squares of two consecutive multiples of 5 is not a multiple of 10.

[2 marks]

Answer

Turn over for next question

6

6(a) Show algebraically that the sum of any 3 consecutive even numbers is always a divisible by 6.

[2 marks]

Answer

6(b) Prove algebraically that $(4n + 2)^2 - (2n + 2)^2$ is a multiple of 4 for all positive integers.

[2 marks]

Answer

6(c) Prove algebraically that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8 for all positive integers of n.

[2 marks]

Answer

Turn over for next question

6

Turn over ►

7(a) If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

[3 marks]

Answer

7(b) Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number.

[2 marks]

Answer

Turn over for next question

5

Turn over ►

8(a) Tom says that $7x - (2x + 3)(x + 2)$ is always negative.

Is he correct? Explain your answer.

[3 marks]

Answer ______________________

8(b) Change a single number in Tom's statement that would lead to a change in your conclusion.

Why is this the case?

[1 mark]

Answer ______________________

4

9(a) Show that the difference between 14^{20} and 21^{2} is a multiple of 7.

[3 marks]

Answer

9(b) Show that $3^{60} - 25$ is not a prime number.

[2 marks]

Answer

Turn over for next question

5

10 Part of a 10×10 $1 - 100$ number grid is pictured below

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

A 2×2 square of numbers is selected.

The following operation is performed:

Difference of the leading diagonal × Difference of the other diagonal

$$(23 - 12) \times (22 - 13) = 11 \times 9 = 99$$

10(a) Verify that this is also the case for a different 2×2 square of numbers on the grid.

[1 mark]

Answer

10(b) Prove this result for all possible 2×2 squares on the grid.

[3 marks]

Answer

End of Questions

4

END