A-LEVEL
Mathematics
MPC4 – Pure Core 4
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$19x-3=A(3-4x)+B(1+2x)$ **Correct** equation and 'attempt' to find A **or** B	M1		e.g. Using $x=\frac{3}{4}$ or $-\frac{1}{2}$ or simultaneous equation such as $19=-4A+2B$ and $-3=3A+B$
	$A=-\frac{5}{2}$	A1		
	$B=\frac{9}{2}$	A1	3	
	NMS or cover up rule scores **SC2** for $A=-\frac{5}{2}$ **or** $B=\frac{9}{2}$ or **SC3** for **both** $A=-\frac{5}{2}$ **and** $B=\frac{9}{2}$			
(b)(i)	$(1+2x)^{-1}=1-2x+kx^2$	M1		provided $k\neq 0$
	$=1-2x+4x^2$	A1		Accept $+(2x)^2$
	$(3-4x)^{-1}=3^{-1}\left(1-\frac{4}{3}x\right)^{-1}$	B1		ACF for 3^{-1} eg $\frac{1}{3}$ or 0.33 …or $0.\dot{3}$
	$\left(1-\frac{4}{3}x\right)^{-1}$ $=1+(-1)\left(-\frac{4}{3}x\right)+\frac{(-1)(-2)}{2!}\left(-\frac{4}{3}x\right)^2$	M1		Condone poor use of or missing brackets.
	$=\left(1+\frac{4}{3}x+\frac{16}{9}x^2\right)$	A1		PI by later work
	$\frac{19x-3}{(1+2x)(3-4x)}=$ $-\frac{5}{2}(1-2x+4x^2)+\frac{9}{2}.\frac{1}{3}\left(1+\frac{4}{3}x+\frac{16}{9}x^2\right)$	M1		PI by correct answer ft on candidate's A and B with their relevant series.
	$=-1+7x-\frac{22}{3}x^2$	A1	7	**Must** have $\frac{22}{3}, 7\frac{1}{3}$ or $7.\dot{3}$
	Alt. $(3-4x)^{-1}=3^{-1}+(-1)3^{-2}(-4x)+\frac{(-1)(-2)}{2!}3^{-3}(-4x)^2$ **M1** $=\frac{1}{3}+\frac{4}{9}x+\frac{16}{27}x^2$ **A2** If A and B are correct full expansions are $-\frac{5}{2}+5x-10x^2$ and $\frac{3}{2}+2x+\frac{8}{3}x^2$ Alt. for combined expansions without using PF's. $\frac{19x-3}{(1+2x)(3-4x)}=(19x-3)(1+2x)^{-1}(3-4x)^{-1}=(19x-3)(1-2x+4x^2)\left(\frac{1}{3}+\frac{4}{9}x+\frac{16}{27}x^2\right)$ Attempt to multiply **any two** of their three brackets together as far as the term in x^2 **M1** then **A1** Condone unsimplified fractions in the binomial expansion(s), but final answer must be fully simplified. A and B could be included in the series – e.g. $\frac{9}{2(3-4x)}=\frac{9}{6-8x}=\frac{9}{6}\left(1-\frac{8x}{6}\right)^{-1}$ could still score **B1 M1 A1**.			
(b)(ii)	$-\frac{1}{2}<x<\frac{1}{2}$ **only**	B1	1	**Strictly** $<$: Accept $\lvert x\rvert<\frac{1}{2}$
	Do **NOT** accept $-1<2x<1$ or $\lvert 2x\rvert<1$ or inclusion of the equality sign. If $-\frac{3}{4}<x<\frac{3}{4}$ is also seen, it must be clear that $-\frac{1}{2}<x<\frac{1}{2}$ is **the** answer.			
	Total		11	

Q2	Solution	Mark	Total	Comment
	$\cos 2\theta = 2\cos^2\theta - 1$ **used**	**B1**		PI: Correct expression in terms of $\cos\theta$ used.
	$3(2\cos^2\theta - 1) - 5\cos\theta + 2 \quad (= 0)$	**M1**		Attempt to use identity for $\cos 2\theta$ of the form $a\cos^2\theta + b$ to obtain a quadratic in $\cos\theta$.
	$6\cos^2\theta - 5\cos\theta - 1 = 0$ $(\cos\theta - 1)(6\cos\theta + 1) = 0$	**m1**		Attempt to factorise their quadratic or correct use of quadratic formula.
	$(\cos\theta = 1) \quad \cos\theta = -\frac{1}{6}$			
	$\theta = 99.6^0 \quad , \ 260.4^0$	**A1** **A1**		**Either** correct – CAO **Both** correct and no extra values in the interval but ignore any values outside of the interval including 0^0 and 360^0.
	Total		5	
	To earn the **m1** mark, **c**andidate's factors must give **their** $6\cos^2\theta$ and **their** -1 i.e. the first and last terms of **their** quadratic. If the quadratic formula is used it must be used correctly for **their** quadratic. If they get the **correct** quadratic and **NMS**, **both** correct answers for $\cos\theta$ implies the **m1** mark, or If they get the **correct** quadratic and **NMS**, **one** correct answer for θ implies the **m1** mark. If they get a **wrong** quadratic, they **must** show the working to (possibly) score the **m1** mark . Interval is specified to be $0^0 < \theta < 360^0$; hence the reason for 'ignoring' solutions 0^0 and 360^0 that might come from$cos\theta = 1$. Degree signs are not required; 99.6 and 260.4 are sufficient for the A marks. Allow **SC1** if both rounded **correctly** to greater accuracy 99.5940… and 260.4059… if **A0 A0**.			

Q3	Solution	Mark	Total	Comment
(a)	$\frac{3+13x-6x^2}{2x-3} = Ax + B + \frac{C}{2x-3}$ $3+13x-6x^2 = Ax(2x-3) + B(2x-3) + C$ Correct above equation and attempt to find one of A, B or C or an attempt at long division	M1		e.g. using $x = \frac{3}{2}$ in an attempt to find C or forming simultaneous equations and attempt to solve.
	$A = -3$	A1		
	$B = 2$	A1		
	$C = 9$	A1	4	
	If long division is used award **M1** once $-3x + \cdots$ has been obtained but only award the **A** marks once the values are clearly identified or it is written in the required form of $Ax + B + \frac{C}{2x-3}$. Alternative method of division $\frac{3+13x-6x^2}{2x-3} = \frac{-3x(2x-3)-9x+13x+3}{2x-3} = -3x + \frac{4x+3}{2x-3}$ **M1** for $-3x + \cdots$. For the **A** marks, A, B and C must be clearly identified or seen in the required form of $Ax + B + \frac{C}{2x-3}$. **NMS** scores **B2** for one correct value, **B3** for 2 correct values and **B4** for all three correct values			
(b)	$\int \frac{3+13x-6x^2}{2x-3}\,dx = \int -3x + 2 + \frac{9}{2x-3}\,dx$ $= px^2 + qx + r\ln(2x-3)$	M1		
	$= -\frac{3}{2}x^2 + 2x + \frac{9}{2}\ln(2x-3)$	A1ft		$\frac{A}{2}x^2 + Bx + \frac{C}{2}\ln(2x-3)$
	Correct **use** of $F(6) - F(3)$ $= \left[-\frac{3}{2}.6^2 + 2.6 + \frac{9}{2}\ln(12-3)\right]$ $-\left[-\frac{3}{2}.3^2 + 2.3 + \frac{9}{2}\ln(6-3)\right]$	m1		**Correct** substitution of limits for their p, q and r.
	$= -\frac{69}{2} + \frac{9}{2}\ln 3$	A1	4	OE
	The **M1 A1ft** and **m1** can be earned even if left in terms of A, B and C or if ‘invented’ value(s) for A, B and C are used. Condone missing brackets from the $\ln(2x-3)$ term for the **M1** mark but **only** award the **A1ft** mark if they have clearly recovered; PI by sight of ln9 or ln3 after using the limits or a correct final answer. Treat a decimal answer (should be−29.55 …) after a correct exact form as ISW but award **A0** if an **exact** answer is not seen.			
	Total		8	

Q4	Solution	Mark	Total	Comment
(a)(i)	$m = m_0 k^t$ Using $m = 12, m_0 = 24$ and $t = 8$ $k^8 = \frac{1}{2}$ or $k = \left(\sqrt[8]{0.5}\right)$ $= 0.917004$	B1	1	$12 = 24k^8$ OE e.g. $k = \left(\frac{1}{2}\right)^{\frac{1}{8}}$ Must see a correct exact expression for k or k^8 or k=0.91700404(32….) to at least 8 d. p. AG be convinced
	Note that **AG** so to earn the mark they must show us a correct exact expression for k or k^8. Accept such as $k = e^{\left(\frac{ln0.5}{8}\right)}$ or $e^{-0.086643\ldots}$ or $\left(\frac{1}{2}\right)^{\frac{1}{8}}$ or $0.91700404(32\ldots)$ as sufficient evidence but withhold the mark if a clear error has been made – e.g. $k = \sqrt[\frac{1}{8}]{0.5}$. Candidates who work with logs must reach an expression such as $\log k = \frac{\log 12 - \log 24}{8}$ first.			
(a)(ii)	$1 = m_0(0.917004)^{60}$ $m_0 = 181$	M1 A1	 2	or $m_0 = (0.917004)^{-60}$ PI by **A1** later Must be 181 no **ISW**
	NMS scores **SC2** for 181 **only** but sight of greater accuracy (181.0198…) implies **M1** if 181 not seen.			
(b)	$m = m_0 k^t$ $8.106 = 10 \times k^{100}$ $k = \sqrt[100]{0.8106}$ OE $\frac{1}{2}m_0 = m_0 k^t$ $k^t = \frac{1}{2}$ $t \log k = \log\left(\frac{1}{2}\right)$ $t = \frac{\log\left(\frac{1}{2}\right)}{\log k}$ $= 330$	M1 A1 M1 A1	 4	 OE: e.g. $k = e^{\ln(0.8106)/100}$ A linear equation in t from $k^t = \frac{1}{2}$ e.g. $t = log_k(0.5)$ Must be 330 No **ISW**
	For guidance, for first **A1**, $k = 0.9979\ldots$ **PI** by later correct work. The first **M1** is for a correct interpretation of the information given so could equally be awarded for an expression involving logs of k such as $\ln 8.106 = \ln 10 + 100 \ln k$ then **A1** for a correct expression for lnk such as $\ln k = \frac{\ln 8.106 - \ln 10}{100}$ or, using base 10, $\log k = \frac{\log 8.106 - 1}{100}$. Those who use the value of k from (a) could only score **M0 A0 M1 A0**. **NMS** scores **SC4** for 330 **only** but sight of greater accuracy (330.1006…) implies **M1 A1 M1** if 330 not seen.			
	Total		**7**	

Q5	Solution	Mark	Total	Comment
(a)(i)	Use of $\sin^2 B + \cos^2 B = 1$ $\left(\frac{1}{\sqrt{5}}\right)^2 + \cos^2 B = 1$ $\cos B = \frac{2}{\sqrt{5}}$	B1	1	Or use of right-angled triangle with opp $= 1$ and hyp $= \sqrt{5}$ to get adj $= \sqrt{4}$ or 2. $\cos\left(\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}}$ is **B0** **AG** ; must see evidence of working
(a)(ii)	$(\sin 2B = 2\sin B\cos B)$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	M1 A1	2	Correct identity (PI) **and** substitution **AG** so line above must be seen.
(b)(i)	$\cos A = \frac{2}{3}$ **exact** value $\sin(A - B) = \sin A\cos B - \cos A\sin B$ $= \frac{\sqrt{5}}{3} \times \frac{2}{\sqrt{5}} - \frac{2}{3} \times \frac{1}{\sqrt{5}}$ Use of $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ or $\frac{10-2\sqrt{5}}{15}$ OE seen $\frac{2}{15}(5 - \sqrt{5})$	B1 M1 m1 A1	4	$\cos A = \frac{2}{3}$ seen or used (**not** 0.667 etc.) ft on their value of cosA $\frac{2}{3\sqrt{5}}$ term becoming $\frac{2\sqrt{5}}{15}$ **before** final answer $\frac{2}{15}$ OE seen and be convinced
	You must see justification between the use of the identity and the final answer to earn the **m1 A1**.			
(b)(ii)	$\cos(A - B) = \cos A\cos B + \sin A\sin B$ $= \frac{2}{3} \times \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3} \times \frac{1}{\sqrt{5}}$ $= \frac{1}{3} + \frac{4}{15}\sqrt{5}$	M1 A1 A1	3	ft on their value of $\cos A$ fully correct OE for $\frac{1}{3}$ and $\frac{4}{15}$ but not left as $\frac{5+4\sqrt{5}}{15}$
	Total		**10**	

Q6	Solution	Mark	Total	Comment
	NO MISREADS ARE ALLOWED IN THIS QUESTION			
(a)	$\overrightarrow{\boldsymbol{AB}} = \begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$	B1		or $\overrightarrow{\boldsymbol{BA}} = \begin{bmatrix} -4 \\ 12 \\ 20 \end{bmatrix}$
	$\overrightarrow{\boldsymbol{AB}}.\begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} = (4 \times 3) + (-12 \times -5) + (-20 \times 1)$	B1ft		Correctly finding scalar product using their $\overrightarrow{\boldsymbol{AB}}$ and direction vector of l_2 Accept $12 + 60 - 20$ or 52
	$52 = \sqrt{4^2 + 12^2 + 20^2}\sqrt{3^2 + 5^2 + 1^2}\cos\theta$	M1		Correct use of their $\boldsymbol{a}.\boldsymbol{b} =$ a b cosθ
	$\cos\theta = \dfrac{52}{\sqrt{560}\sqrt{35}}$	A1		OE all correct in this form or better.
	$= \frac{13}{35}$	A1	5	CAO
	The **B1** mark for $\overrightarrow{\boldsymbol{AB}}$ or $\overrightarrow{\boldsymbol{BA}}$ or any multiple could be **PI** by its use in the scalar product e.g. $\begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$ etc. The **B1ft** mark is for the scalar product of their $\overrightarrow{\boldsymbol{AB}}$ with the direction vector of l_2. The **M1** mark is for a clear attempt at the scalar product definition of "$\boldsymbol{a}.\boldsymbol{b} = a\ b\cos\theta$" in any form using their $\overrightarrow{\boldsymbol{AB}}$ with the direction vector of l_2 . As in the MS, there is no need for minus signs in squared terms. Provided they earn the **M1** mark with the correct values included, it is possible to score **both A1** marks for $\cos\theta = \frac{13}{35}$ without the intermediate form being seen.			
(b)	Line AB: $(\boldsymbol{r}\ =)\ \begin{bmatrix} 0 \\ 6 \\ 9 \end{bmatrix} + \mu\begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$			
	$-1 + 3\lambda = 0 + 4\mu$ $5 - 5\lambda = 6 - 12\mu$ $-2 + \lambda = 9 - 20\mu$	M1		Set up two correct equations for their l_1 but with correct l_2 **and** attempt to eliminate λ or μ. Accept these three equations in column vector form
	$\lambda = 1 \quad \mu = \dfrac{1}{2}$	A1A1		
	Verifying that all 3 three equations are satisfied **and** conclusion – e.g. 'intersect'.	E1		Clear checking of λ and μ in unused equation or showing P lies on both lines. Dependent on **correct** co-ordinates of P.
	$P(2, 0, -1)$	A1	5	Accept as a column vector
	Look out for any alternative correct versions for the vector equation of l_1, e.g. If $B(4, -6, -11)$ is used as known point in l_1 this leads to $\lambda = 1$ and $\mu = -\frac{1}{2}$ but also look out for 'multiples' of $\begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$ being used as the direction vector.			

Q6	Solution	Mark	Total	Comment
(c)	Using points C and D			
	$\overrightarrow{CD} = t\begin{bmatrix}3\\-5\\1\end{bmatrix}$	B1		Since C and D lie on l_2
	$\lvert\overrightarrow{AB}\rvert = 3\,\lvert\overrightarrow{CD}\rvert \quad \Rightarrow \quad \sqrt{560} = 3\sqrt{35t^2}$			or $\sqrt{560} = 3t\sqrt{35}$
	$t = (\pm)\ \frac{4}{3}$	B1		$\frac{4}{3}$ seen
	$(\ \overrightarrow{OC/D}\ ) = \overrightarrow{OP} + \frac{t}{2}\begin{bmatrix}3\\-5\\1\end{bmatrix}$			Uses P the mid-point of AB and CD in this method.
	$= \begin{bmatrix}2\\0\\-1\end{bmatrix} + \frac{2}{3}\begin{bmatrix}3\\-5\\1\end{bmatrix}$ **or** $\begin{bmatrix}2\\0\\-1\end{bmatrix} - \frac{2}{3}\begin{bmatrix}3\\-5\\1\end{bmatrix}$	M1		Use of their $\overrightarrow{OP}$ and $\frac{t}{2}\begin{bmatrix}3\\-5\\1\end{bmatrix}$
	$\left(4\,, -\frac{10}{3}\,, -\frac{1}{3}\right)$	A1		Accept as a column vector.
	$\left(0\,, \frac{10}{3}\,, -\frac{5}{3}\right)$	A1	5	Accept as a column vector.
	Candidate could just get $t = \frac{4}{3}$ giving point $\left(4\,, -\frac{10}{3}\,, -\frac{1}{3}\right)$ and then find $\left(0\,, \frac{10}{3}\,, -\frac{5}{3}\right)$ by symmetry.			
(c)	R is any point on l_2			
	$(\overrightarrow{OR}) = \begin{bmatrix}-1+3\lambda\\5-5\lambda\\-2+\lambda\end{bmatrix}$			
	$(\overrightarrow{PR}) = \begin{bmatrix}-1+3\lambda\\5-5\lambda\\-2+\lambda\end{bmatrix} - \begin{bmatrix}2\\0\\-1\end{bmatrix}$	(B1)		$\begin{bmatrix}-3+3\lambda\\5-5\lambda\\-1+\lambda\end{bmatrix}$ but award at $\overrightarrow{OR} - \overrightarrow{OP}$ stage
	If R is at C or D then $\lvert\overrightarrow{AB}\rvert = 3\lvert\overrightarrow{CD}\rvert = 6\lvert\overrightarrow{PR}\rvert$			
	$\sqrt{560}=6\sqrt{(-3+3\lambda)^2+(5-5\lambda)^2+(-1+\lambda)^2}$			
	$560 = 36(35\lambda^2 - 70\lambda + 35)$	(B1)		or better
	$9\lambda^2 - 18\lambda + 5 = 0$			
	$(3\lambda - 1)(3\lambda - 5) = 0$	(M1)		Factorising (their $9\lambda^2$ and $+5$ terms) or attempting to solve their quadratic equation correctly – PI by correct values of λ.
	$\lambda = \frac{1}{3}$ **or** $\frac{5}{3}$			
	$\left(0\,, \frac{10}{3}\,, -\frac{5}{3}\right)$	(A1)		Accept as a column vector.
	$\left(4\,, -\frac{10}{3}\,, -\frac{1}{3}\right)$	(A1)	(5)	Accept as a column vector.
	Total		**15**	

Q7	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt}\right) = -(-6)\frac{e^{2-6t}}{4}$	B1		ACF : $\frac{3}{2}e^{2-6t}$
	$\left(\frac{dy}{dt}\right) = \frac{pe^{3t}.t + qe^{3t}}{(3t)^2}$	M1		From quotient rule
	$= \frac{3e^{3t}.3t - e^{3t}.3}{(3t)^2}$	A1		ACF : $\frac{9te^{3t}-3e^{3t}}{9t^2}$, $\frac{e^{3t}}{t} - \frac{e^{3t}}{3t^2}$ etc.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9.\frac{2}{3}.e^2 - 3e^2)/4}{\frac{3}{2}e^{-2}}$	m1		Using $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} / \frac{\mathrm{d}x}{\mathrm{d}t}$ **and** clear evidence of an attempt to substitute $t = \frac{2}{3}$ (**must** be this) done in either order.
	$= \frac{1}{2}e^4$	A1	5	**CAO :** Accept this or $\frac{e^4}{2}$ or $0.5e^4$ **only**
	$\frac{\mathrm{d}y}{\mathrm{d}t}$ found using product rule : $\frac{\mathrm{d}y}{\mathrm{d}t} = pe^{3t}.t^{-1} + qe^{3t}.t^{-2}$ **M1** $= e^{3t}t^{-1} - \frac{1}{3}e^{3t}t^{-2}$ **A1** $\left(= \frac{e^{3t}}{t} - \frac{e^{3t}}{3t^2}\right)$ For guidance, when $t = \frac{2}{3}$, $\frac{dx}{dt} = \frac{3}{2}e^{-2}$ and $\frac{dy}{dt} = \frac{3}{4}e^2$			
(b)	$x = \frac{4-e^{2-6t}}{4} \Rightarrow 4x = 4 - e^{2-6t}$ leading to			
	$e^{-6t} = \frac{4-4x}{e^2}$	M1		Any correct expression for e^{-6t}or e^{6t}.
	$e^{6t} = \frac{e^2}{4(1-x)}$			
	$e^{3t} = \frac{e}{2\sqrt{1-x}}$	A1	2	**AG** Must see inversion step and be convinced they haven't worked backwards
	Alternative for the **M1** mark is a correct expression for $3t$ or e^{1-3t} or $\frac{1}{2}e^{1-3t}$ OE. $4x = 4 - e^{2-6t} \Rightarrow 2 - 6t = \ln(4-4x) \Rightarrow 3t = 1 - \frac{1}{2}\ln(4-4x)$ **M1** or $4x = 4 - e^{2-6t} \Rightarrow e^{2-6t} = 4 - 4x \Rightarrow e^{1-3t} = \sqrt{4-4x}$ OE e.g. $\frac{e}{2e^{3t}} = \sqrt{1-x}$ (OE) for **M1**, then further **correct** working needed before printed answer for **A1**.			
(c)	From (b) $e^{3t} = \frac{e}{2\sqrt{1-x}}$			
	$\ln(e^{3t}) = \ln\left(\frac{e}{2\sqrt{1-x}}\right)$			
	$3t = \ln e - \ln(2\sqrt{1-x})$	M1		Find $3t$ (or t) in terms of x; must have used laws of logs correctly on both sides (possibly $\ln e = 1$ at this stage)
	$y = \frac{e}{2\sqrt{1-x}\left[1 - \ln 2\sqrt{1-x}\right]}$	A1	2	From $y = \frac{e^{3t}}{3t}$; must be in this form
	Alt. From x, $t = \frac{1}{6}(2 - \ln(4-4x))$ **M1** or an expression for 3t then **A1** for printed answer.			
	Total		9	

Q8	Solution	Mark	Total	Comment
(a)	$\theta = \tan^{-1}\left(\frac{3x}{2}\right) \Rightarrow 2\tan\theta = 3x$			
	$2\sec^2\theta \,.\dfrac{d\theta}{dx} = 3$	B1		or $2\sec^2\theta = 3\frac{dx}{d\theta}$
	$\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{3x}{2}\right)^2$	M1		Use of correct identity to get $\sec^2\theta$ in terms of x ; condone missing bracket.
	$\frac{d\theta}{dx} = \frac{3}{2\left(1+\frac{9x^2}{4}\right)} = \frac{6}{4+9x^2}$	A1	3	**Correct** algebra with $k = 6$. $\frac{k}{4+9x^2}$ **is given** so be convinced
	If implicit differentiation **not** used then 0/3. Embedded answer of $k = 6$ is sufficient. The **M1** and **A1** marks are available even if the candidate has $\frac{dy}{dx}$ as their derivative.			
(b)	$9y(4+9x^2)\dfrac{dy}{dx} = \text{cosec } 3y$			
	$\int 9y \sin 3y \; dy = \int \frac{1}{4+9x^2}\, dx$	B1		**Correct** separation including dy and dx at 'ends' of integrals (do not penalise unless the dy or dx is directly under the fraction bar), the integral signs at the 'front' and the integrands. The 9 can be on RHS as $\frac{1}{9}$.
	LHS : parts with $u = (9)y$ and $dv = \sin 3y$	M1		**M1** for $p\, y\cos 3y + q\int \cos 3y\, dy$
	$(9)\left(-\frac{1}{3}\, y\cos 3y + \int \frac{1}{3}\cos 3y\, dy\right)$	A1		
	$-3y\cos 3y + \sin 3y$	A1		If 9 used on the LHS
	RHS: $\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$	B1ft		ft on k from (a) i.e. $\frac{1}{k}\tan^{-1}\left(\frac{3x}{2}\right)$.
	$-3y\cos 3y + \sin 3y = \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + c$			
	Using $x = 0$ and $y = \frac{\pi}{3}$ to find c $\sin\pi - 3.\frac{\pi}{3}.\cos\pi = 0 + c$ gives $c = \pi$	M1		Must have an expression of form $p\sin 3y + q y\cos 3y = r\tan^{-1}\left(\frac{3x}{2}\right) + c$ and use x=0 and $y = \frac{\pi}{3}$ to find c. PI by a correct **ft** value for c.
	$-3y\cos 3y + \sin 3y = \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + \pi$	A1	7	OE but must be a complete, correct expression
	Correct separation must be seen **on a single line** but accept $\int \frac{9y}{\text{cosec}3y}\, dy$ on LHS. With the 9 on the RHS, the indefinite integral line should read $\frac{1}{9}\sin 3y - \frac{1}{3}\, y\cos 3y = \frac{1}{54}\tan^{-1}\left(\frac{3x}{2}\right) + k$.			
	Total		**10**	