Please write clearly in block capitals.

| Centre number | | | | | | Candidate number | | | | |
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Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

# A-level
# MATHEMATICS
## Unit Pure Core 4

Friday 15 June 2018 Afternoon Time allowed: 1 hour 30 minutes

**Materials**
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
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| TOTAL | |

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Answer **all** questions.

Answer each question in the space provided for that question.

**1 (a)** Express $\frac{10 + 24x - 12x^2}{(3-x)(1+4x)}$ in the form $A + \frac{B}{3-x} + \frac{C}{1+4x}$, where $A$, $B$ and $C$ are integers.

**[4 marks]**

**(b)** Hence find $\int_0^2 \frac{10 + 24x - 12x^2}{(3-x)(1+4x)}\, dx$, giving your answer in the form $p + q \ln 3$.

**[5 marks]**

QUESTION PART REFERENCE

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**2** The angle $\alpha$ is **acute** and $\cos\alpha = \frac{\sqrt{3}}{3}$. The angle $\beta$ is **obtuse** and $\sin\beta = \frac{1}{3}$.

**(a)** Show that $\tan\alpha = \sqrt{2}$ and find an exact value for $\tan\beta$.

**[3 marks]**

**(b)** Hence show that $\tan(\alpha - \beta)$ can be written as $p\sqrt{2}$, where $p$ is a rational number.

**[2 marks]**

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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**3 (a)** Find the binomial expansion of $(1-9x)^{\frac{2}{3}}$ up to and including the term in $x^2$.

**[2 marks]**

**(b) (i)** Find the binomial expansion of $(64-9x)^{\frac{2}{3}}$ up to and including the term in $x^2$.

**[3 marks]**

**(ii)** Use your expansion from part **(b)(i)** to find an estimate for $67^{\frac{2}{3}}$, giving your answer in the form $p+\frac{q}{r}$ where $p$, $q$ and $r$ are positive integers with $q<r$.

**[2 marks]**

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**4** The polynomial $f(x)$ is defined by $f(x) = 18x^3 - 3x^2 - 28x - 12$.

**(a) (i)** Use the Factor Theorem to show that $(3x + 2)$ is a factor of $f(x)$.

**[2 marks]**

**(ii)** Express $f(x)$ as a product of linear factors.

**[3 marks]**

**(b)** The function g is defined for all real values of $\theta$ by

$$g(\theta) = 18\sin 2\theta\cos\theta - 3\cos 2\theta + 20\sin\theta + 27$$

**(i)** Show that the equation $g(\theta) = 0$ can be written as $f(x) = 0$, where $x = \sin\theta$.

**[4 marks]**

**(ii)** Hence solve the equation $g(\theta) = 0$, giving your answers, in radians, to three significant figures in the interval $0 \leqslant \theta \leqslant 2\pi$.

**[2 marks]**

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**5** In a conservation area, a disease is spreading amongst two species of wild animal, $P$ and $Q$, which is reducing their numbers.

Previous experience has shown that the number of each of the species $P$ and $Q$ can be modelled by

$$\mathrm{p}(t) = 4500\mathrm{e}^{-\frac{1}{20}t} \quad \text{and} \quad \mathrm{q}(t) = 3000\mathrm{e}^{-\frac{1}{40}t} \text{ respectively}$$

where $t$ is the time in weeks after the disease is first detected.

This outbreak of the disease was first detected on 1 May.

**(a)** Use the two models to find:

**(i)** the number of species $P$ on 1 May;

**[1 mark]**

**(ii)** the number of species $Q$ after $36$ weeks from 1 May, giving your answer to the nearest $10$;

**[1 mark]**

**(iii)** after how many weeks the number of species $P$ will first fall below $1500$.

**[2 marks]**

**(b)** Use logarithms and the two models to calculate the value of $t$ when the number of species $Q$ will be four times that of species $P$. Give your answer to the nearest whole number.

**[3 marks]**

**(c)** When $t = T$ the number of species $Q$ first exceeds that of species $P$ by $300$.

**(i)** Use this information and the two models to derive a quadratic equation in $x$ where $x = \mathrm{e}^{-\frac{1}{40}T}$.

**[2 marks]**

**(ii)** Hence find the number of days after 1 May when this difference of $300$ animals will first occur. Give your answer to the nearest day.

**[3 marks]**

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**6** A curve is defined by the equation

$$\cos 3y + y\sin^2 3x = x + k$$

The point $P\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ lies on this curve.

**(a)** Find the exact value of the constant $k$.

**[1 mark]**

**(b)** Find an expression for $\frac{dy}{dx}$.

**[6 marks]**

**(c)** Find the equation of the tangent to the curve at $P$, giving your answer in the form $y = mx + c$.

**[2 marks]**

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**7 (a)** Using a suitable substitution, or otherwise, find

$$\int \frac{x}{(7+2x^2)^2}\,\mathrm{d}x$$

**[3 marks]**

**(b)** Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x\,\mathrm{e}^{4y}}{(7+2x^2)^2}$$

given that $y = 0$ when $x = 2$.

Give your answer in the form $y = \mathrm{f}(x)$.

**[6 marks]**

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**8** The points $A$ and $C$ have coordinates $(3, -1, 2)$ and $(0, -1, -2)$ respectively.

The line $l$ has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$.

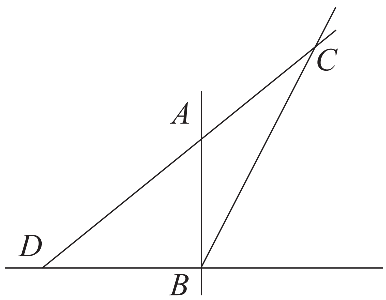
**(a) (i)** The point $B$ lies on $l$ where $\lambda = 2$. Find the coordinates of $B$.

**[1 mark]**

**(ii)** Find the **acute** angle $ABC$, giving your answer to the nearest $0.1°$.

**[5 marks]**

**(b)** The point $D$ lies on a line through $C$ and $A$ such that angle $ABD$ is a right angle.




The point $E$ completes the rectangle $ABDE$. Find the coordinates of $E$.

**[7 marks]**

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