**General Certificate of Education
Advanced Subsidiary Examination
June 2010**

Mathematics   MPC1

Unit Pure Core 1

Monday 24 May 2010   1.30 pm to 3.00 pm

For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You must **not** use a calculator.

Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer the questions in the spaces provided. Do not write outside the box around each page.
• Show all necessary working; otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to be marked.
• The use of calculators is **not** permitted.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.
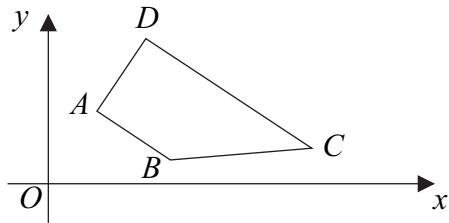
Answer **all** questions in the spaces provided.

1 The trapezium *ABCD* is shown below.

The line *AB* has equation $2x + 3y = 14$ and *DC* is parallel to *AB*.

(a) Find the gradient of *AB*. *(2 marks)*

(b) The point *D* has coordinates $(3, 7)$.

    (i) Find an equation of the line *DC*. *(2 marks)*

    (ii) The angle *BAD* is a right angle. Find an equation of the line *AD*, giving your answer in the form $mx + ny + p = 0$, where *m*, *n* and *p* are integers. *(4 marks)*

(c) The line *BC* has equation $5y - x = 6$. Find the coordinates of *B*. *(3 marks)*

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2 (a) Express $\left(3 - \sqrt{5}\right)^2$ in the form $m + n\sqrt{5}$, where m and n are integers. *(2 marks)*

(b) Hence express $\dfrac{\left(3 - \sqrt{5}\right)^2}{1 + \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

(4 marks)

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3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 2$. *(2 marks)*
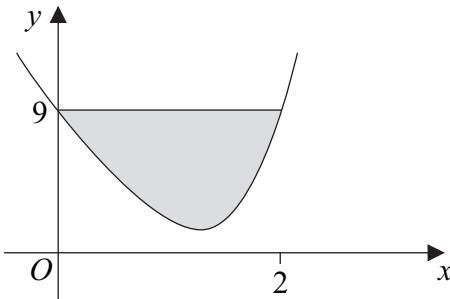
(c) (i) Verify that $p(-1) < p(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

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4   The curve with equation $y = x^4 - 8x + 9$ is sketched below.

The point $(2, 9)$ lies on the curve.

(a) (i)   Find $\displaystyle\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

(ii)   Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

(b)   The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i)   Find the gradient of the curve at the point A. *(4 marks)*

(ii)   Hence find an equation of the tangent to the curve at the point A. *(1 mark)*
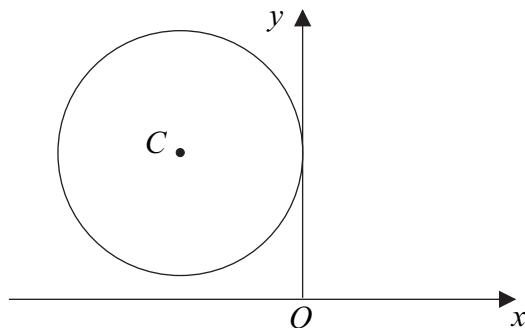
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5   A circle with centre $C(-5, 6)$ touches the y-axis, as shown in the diagram.

(a)   Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

(3 marks)

(b) (i)   Verify that the point $P(-2, 2)$ lies on the circle.   *(1 mark)*

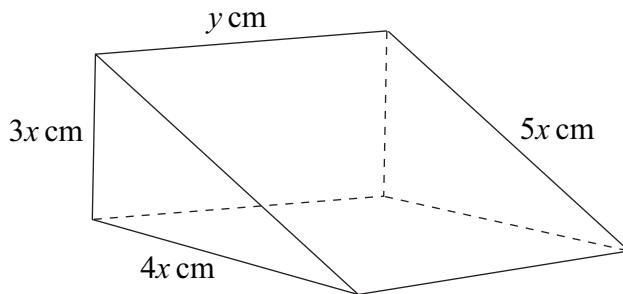**(ii)**   Find an equation of the normal to the circle at the point P.   *(3 marks)*

(iii)   The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O.   *(4 marks)*

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6   The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.

The total surface area of the five faces is $144\,\text{cm}^2$.

(a) (i)   Show that $xy + x^2 = 12$. *(3 marks)*

(ii)   Hence show that the volume of the block, $V\,\text{cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i)   Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii)   Show that V has a stationary value when $x = 2$. *(2 marks)*

(c)   Find $\dfrac{\mathrm{d}^2 V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

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7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers.

(2 marks)

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots.

(2 marks)

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$.

(4 marks)

(ii) Hence find the possible values of k.

(4 marks)

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END OF QUESTIONS

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