Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Further Mathematics
AS Further Decision D2 Paper 8FM0_28

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	Reducing rows and columns $\begin{array}{cc} & \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 4 & 0 & 4.5 & 6 \\ 3 & 0 & 5 & 5.5 \\ 0.5 & 0 & 0 & 4.5 \\ 2.5 & 1.5 & 0 & 0.5 \end{pmatrix} \end{array}$ $\begin{array}{cc} & \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 3.5 & 0 & 4.5 & 5.5 \\ 2.5 & 0 & 5 & 5 \\ 0 & 0 & 0 & 4 \\ 2 & 1.5 & 0 & 0 \end{pmatrix} \end{array}$	M1 A1	1.1b 1.1b
	augment by 2.5 $\begin{array}{cc} & \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 0 & 2.5 & 2.5 \\ 0 & 2.5 & 0 & 4 \\ 2 & 4 & 0 & 0 \end{pmatrix} \end{array}$	M1 A1ft	1.1b 1.1b
	A – Q, B – P, C – R, D – S	A1ft	2.2a
			(5 marks)

Notes

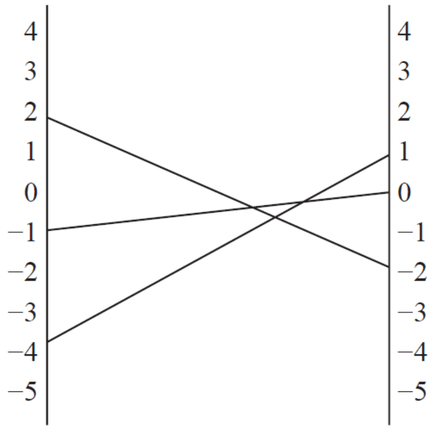
M1: simplifying the initial matrix by reducing rows and then columns

A1: cao

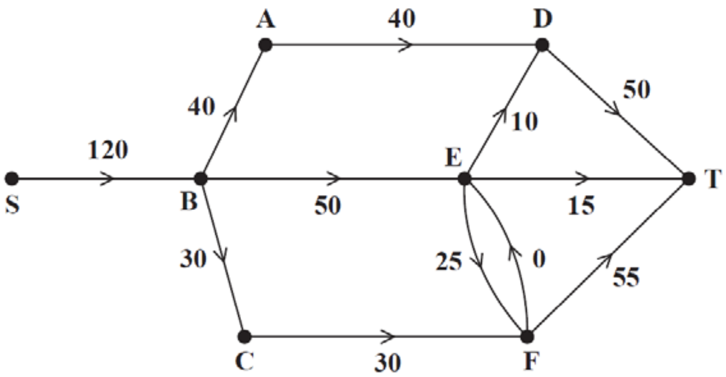
M1: develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines to 4 lines needed

A1ft: allow follow through from <u>one</u> numerical slip only during row/column reduction

A1ft: dependent on all previous M marks and one A mark – to deduce the optimal allocation from the location of the zeros in the table

Question	Scheme	Marks	AOs
2(a)	The gains (or losses) made by one player are exactly balanced by the losses (or gains) made by the other player.	B1	1.2
		(1)	
(b)	3	B1	1.1b
		(1)	
(c)	e.g. if a member of team *A* gains *x* points then a member of team *B* gains $10 – x$ points. Subtracting 5 from both gives *A*: $x – 5$ and $(10 – x) – 5 = 5 – x$. The sum is $(x – 5) + (5 – x) = 0$	B1	2.4
		(1)	
(d)	**(i)** Row minima: –2, –4, –1 max is –1 Column maxima: 0, 1, 3 min is 0 Play safe for Team *A* is Olive and for Team *B* is Paul **(ii)** Row maximin (–1) $\neq$ Col minimax (0) so not stable	M1 A1 A1 B1	1.1b 1.1b 1.1b 2.4
		(4)	
(e)	If *B* plays strategy 1, *A*'s gains are $–1(1 – p) = p – 1$ If *B* plays strategy 2, *A*'s gains are $p + –4(1 – p) = 5p – 4$ If *B* plays strategy 3, *A*'s gains are $–2p + 2(1 – p) = 2 – 4p$  $2 - 4p = 5p - 4 \Rightarrow p = 2/3$ Team A should play Mischa with probability 2/3 and Noel with probability 1/3	M1 A1 M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b 1.1b 3.2a
		(6)	
(f)	**(i)** 13/3 **(ii)** 17/3	B1 B1ft	1.1b 2.2a
		(2)	
			(15 marks)

Notes
(a) **B1:** cao - indication that either the losses of one (player) are balanced by the gains of the other (player) or that the total points scored by both (players) is zero **(b)** **B1:** cao (3) **(c)** **B1:** correct explanation – could explain their reasoning using a numerical example **(d)(i)** **M1:** finding row minimums and column maximums – condone one error **A1:** max (row minima) and min (column maxima) correct – dependent on all correct values of row minimums and column maximums **A1:** correct play safes for both teams (Olive (O) and Paul (P)) **(d)(ii)** **B1:** row maximin (–1) $\neq$ col minimax (0) (so not stable) **(e)** **M1:** setting up three expressions in terms of p with at least one correct **A1:** all three expressions correct **M1:** axes correct, at least one line correctly drawn for their expressions **A1:** correct graph with consistent scaling (lines must not extend past $p < 0$ and $p > 1$) **A1:** correct probability expressions leading to correct value of p **A1ft:** interpret their value of p in the context of the question – must refer to play and the team members **(f)(i)** **B1:** cao **(f)(ii)** **B1ft:** 10 – their answer to (f)(i) **SC for (d) and (e) for those candidates who do not consider the zero-sum game** For (d) M1 only for finding row minimums (3, 1, 4) and column maximums (5, 6, 8) – condone one error. Then allow possibility of full marks in (e) – expressions should be $p + 4$, $5p + 1$ and $-4p + 7$ and the graph should lead to $5p + 1 = -4p + 7$

Question	Scheme	Marks	AOs
3(a)	**(i)** 170 **(ii)** 145	B1 B1	1.1b 1.1b
		(2)	
(b)	Deduces the maximum possible flow is ≤ 145 litres per minute	B1ft	2.2a
		(1)	
(c)		M1 A1	2.2a 1.1b
		(2)	
(d)	Cut through arcs BA, ED, ET, EF (twice), CF Maximum flow = minimum cut Flow = 120, Cut = 120 therefore flow of 120 is optimal	B1 B1	3.1a 2.1
		(2)	
(e)	$0 < x \leq 25$, flow is $120 + x$ $x > 25$, flow is 145	M1 A1 A1	3.1a 2.2a 2.3
		(3)	
			(10 marks)

Notes
(a) **B1:** cao **B1:** cao **(b)** **B1ft:** deduced from their least value given in (a) - must include 'less than or equal to' **(c)** **M1:** deduces that the flow out of SB must equal 120 and that the 'flow in = flow out' at all but one node **–** one number only required on each arc (condone blank for arc FE) **A1:** a correct valid flow through the network (check that flow in must equal flow out at each vertex) **(d)** **B1:** finds a correct cut through saturated arcs directed from S to T **B1:** correct mathematical argument that the maximum flow is 120 **-** dependent on correct cut and correct flow in (c) – must state 'maximum flow = minimum cut' **(e)** **M1:** understanding that the flow through the system will be different depending on the possible values of x (this could be shown by either of the flows being stated correctly or by consideration of the critical value of $x = 25$) **A1:** correct deduction of both possible flows: $120 + x$ and 145 **A1:** correct argument (in terms of the correct inequalities) for when the flow is valid for $120 + x$ and 145 **SC in (e) –** award M1A1 for one correct flow and interval

Question	Scheme	Marks	AOs
4(a)	$r = 2$ $N = 50$	B1 B1	3.4 1.1b
		(2)	
(b)	(aux equation $m - 1.02 = 0 \Rightarrow$) complementary function is $A(1.02)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 1.02\lambda = 50$ $\Rightarrow \lambda = ...$	M1	1.1b
	General solution is $u_n = A(1.02)^n - 2500$	A1	1.1b
	$n = 1,\ u_1 = 560 \Rightarrow A = ...$	M1	3.4
	$u_n = 3000(1.02)^n - 2500$	A1	1.1b
		(5)	
(c)	$3000(1.02)^n - 2500 > 3000$	M1	1.1b
	$(1.02)^n > \dfrac{11}{6} \Rightarrow n\log(1.02) > \log\left(\dfrac{11}{6}\right)$	M1	1.1b
	$n > 30.6088... \Rightarrow n = 31$	A1	1.1b
		(3)	
			(10 marks)

Notes
(a) **B1:** cao **B1:** cao **(b)** **B1:** cao **M1:** substituting their trial solution into the recurrence relation in an attempt to find their λ **A1:** cao for the general solution **M1:** using the conditions in the model to calculate A **A1:** cao for the particular solution Alternative approach for **(b)** **B1:** $(1.02)^n u_0$ **M1:** attempt sum of GP with a = 50 and r = 1.02 $\left(u_n = \ldots + \dfrac{50(1-1.02^n)}{1-1.02}\right)$ **A1:** general solution is $u_n = (1.02)^n(u_0 + 2500) - 2500$ (or equivalent) **M1:** Uses $u_1 = 560$ to find u_0 (e.g. $560 = 1.02u_0 + 50 \Rightarrow u_0 = \ldots$) **A1:** $u_n = 3000(1.02)^n - 2500$ **(c)** **M1:** sets their particular solution greater than 3000 (condone equals) – their particular solution must be of the correct form $\left(u_n = c(1.02)^n \pm d\right)$ **M1:** dependent on previous M mark – re-arranging and correctly applies the process of taking logs for their particular solution **A1:** cao (allow correct answer to 3 significant figures or 31)