Oxford Cambridge and RSA

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AS Level Further Mathematics B (MEI)

Y414 Numerical Methods

Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- In each question you must show sufficient detail of the method(s) which you are using.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 The numbers x and y are approximated to three significant figures by $X = 4.15$ and $Y = 342$.

(i) Calculate the maximum possible relative error in each of these approximations. **[2]**

(ii) Hence calculate an estimate of the maximum possible relative error in using $\frac{Y}{X}$ as an approximation to $\frac{y}{x}$. **[2]**

2 The following table gives some values of a function $\mathrm{g}(x)$.

x	0	1	2	3	4
$\mathrm{g}(x)$	2	1	a	11	b

(i) Construct a difference table as far as the third differences. **[2]**

(ii) Given that $\mathrm{g}(x)$ is quadratic, find the values of a and b. **[3]**

(iii) Express $\mathrm{g}(x)$ in the form $px^2 + qx + r$, where p, q and r are numbers to be determined. **[3]**

3 It is required to solve the equation $\mathrm{f}(x) = 0$, where $\mathrm{f}(x) = x^5 - 3\sqrt{x} + 1$, by using an iterative method.

(i) Obtain a Newton-Raphson formula for successive iterations. **[3]**

With a starting value of $x_0 = 1$ the following iterates are generated by the Newton-Raphson formula for this equation.

1
1.285 714 286
1.195 628 311
1.177 204 057
1.176 497 074
1.176 496 062
1.176 496 062

(ii) (*A*) Calculate the ratios of differences for these iterates. **[2]**

(*B*) State what the values calculated in part **(ii)** (*A*) tell you about the speed of convergence of the Newton-Raphson iteration in this case. **[1]**

4 In a certain country the annual tax bill for each taxpayer is calculated to the nearest cent. When the data are transferred to the data base at the tax office the software chops each bill to the nearest dollar. There are 10 000 000 taxpayers, and 1 dollar = 100 cents.

(i) The amount which this chopping will lose the government in each tax year is modelled by assuming that, for each taxpayer, the maximum possible is lost. Use this model to calculate the amount which the government loses each tax year. **[2]**

(ii) Explain why the model in part **(i)** is unlikely to give a good estimate of the money lost by the government each tax year. **[1]**

(iii) Calculate the expected loss in one tax year. **[1]**

(iv) Explain the modelling assumption which underlies your calculation in part **(iii)**. **[1]**

5 A spreadsheet is used to calculate $\sum_{r=1}^{12} \frac{1}{r^9}$.

The following spreadsheet output shows the first steps in the process.

	A	B
1	*r*	f(*r*)
2	1	1
3	2	0.001953125
4	3	5.08053E-05
5	4	3.8147E-06
6	5	0.000000512
7	6	9.9229E-08
8	7	2.47809E-08
9	8	7.45058E-09
10	9	2.58117E-09
11	10	0.000000001
12	11	4.24098E-10
13	12	1.93807E-10
14		

Cell B3 contains the formula `=1/(A3^9)`.

(i) State the purpose of this formula. **[1]**

(ii) Write down a formula which can be entered into cell B14 to complete the calculation. **[2]**

When the calculation is completed correctly, the value 1.002 008 393 is displayed in cell B14.

When $\sum_{r=1}^{20} \frac{1}{r^9}$ is calculated in a similar way, the same value, 1.002 008 393, is displayed in a cell.

(iii) Explain why this happens. **[2]**

6 **(i)** (*A*) Sketch the graphs of $y = 2^x$ and $y = 3x + 2$ for $-5 \le x \le 5$ on the same coordinate axes. **[2]**

(*B*) Hence state the number of real roots of the equation $2^x - 3x - 2 = 0$ for $-5 \le x \le 5$. **[1]**

(ii) Use the iteration $x_{r+1} = \dfrac{\left(2^{x_r} - 2\right)}{3}$ with $x_0 = 0$ to find the root near $x = 0$ correct to 5 decimal places. **[3]**

(iii) The iteration $x_{r+1} = \dfrac{\log_{10}\left(3x_r + 2\right)}{\log_{10} 2}$ may be used successfully to find the root near $x = 4$.

A student starts by entering `= 4` in cell A1 of a spreadsheet.

In cell A2 he enters `=LOG10(3A1)+2/LOG10(2)`.

This formula is incorrect. Write down the correct formula needed to obtain the result of the first iteration on the spreadsheet. **[2]**

7 When a 5 kg block of kryptonite is immersed in water it slowly dissolves. The mass of kryptonite remaining undissolved at time *t* hours, *A* kg, is modelled as follows.

- For the first 4 hours $A = 5 \times b^{\sqrt{t}}$ where *b* is a constant.
- Whatever the rate of dissolving is at *t* = 4, the kryptonite then dissolves at this constant rate until there is none left.

(i) After 4 hours the mass of kryptonite remaining is measured as 4.05 kg, correct to the nearest 50 grams. By making suitable calculations obtain the possible range of values for *b*. **[2]**

It is decided to apply the model using *b* = 0.9.

(ii) Use the forward difference method to

- produce an estimate of $\dfrac{\mathrm{d}A}{\mathrm{d}t}$ at *t* = 4 with *h* = 0.01,
- produce an estimate of $\dfrac{\mathrm{d}A}{\mathrm{d}t}$ at *t* = 4 with *h* = 0.001. **[3]**

(iii) Hence write down the best estimate of $\dfrac{\mathrm{d}A}{\mathrm{d}t}$ at *t* = 4 which can be justified from your answers in part **(ii)**. **[1]**

(iv) According to the model, when will all the kryptonite be completely dissolved? **[1]**

(v) The time when the kryptonite is completely dissolved is required to the nearest hour. Comment on whether the model will achieve this. **[1]**

8 The integral $I = \int_0^1 \sqrt{1-x^3}\,dx$ is to be evaluated numerically.

(i) (*A*) Find estimates of *I* by:

- using the midpoint rule with one strip;
- using the trapezium rule with one strip. **[4]**

(*B*) Hence find an estimate of *I* using Simpson's rule. **[2]**

Further approximations to *I* using the midpoint rule and the trapezium rule are found by successively reducing *h* by a factor of 2 as far as $h = 0.0078125$. These approximations are shown in the spreadsheet output below. Some values in the spreadsheet have been deliberately omitted.

	A	B	C	D
1	*h*	Midpoint	Trapezium	Simpson
2	1			
3	0.5	0.876251029	0.717707173	
4	0.25	0.854056168	0.796979101	
5	0.125	0.845891776	0.825517635	
6	0.0625	0.842943197	0.835704705	
7	0.03125	0.841889401	0.839323951	
8	0.015625	0.841514808	0.840606676	
9	0.0078125	0.841382011	0.841060742	

(ii) (*A*) Write down the best possible estimate of *I* which can be justified from these approximations. **[1]**

(*B*) Give a reason for the precision of your estimate in part **(ii)** (*A*). **[1]**

(iii) Give two different spreadsheet formulae which could be used to give the Simpson's rule estimate $S_{64} = 0.841212097$ in cell D8. The formulae should use cell references only from columns B and C. **[4]**

In order to evaluate *I* as accurately as possible, the column of Simpson's rule estimates was completed; the ratio of differences for these estimates was found to converge to approximately 0.353. This value was used to obtain an improved approximation by extrapolating to infinity. A second approximation was found by extrapolating with the theoretical value $r = \left(\frac{1}{2}\right)^4 = 0.0625$. The following results were obtained.

Extrapolation with $r = 0.353$	Extrapolation with $r = 0.0625$
0.841 309 2	0.841 279 1

(iv) State which of these values is more reliable and justify your answer. **[2]**

(v) Obtain the best possible estimate of *I*. You should explain your reasoning carefully. **[2]**

END OF QUESTION PAPER

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Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

AS Level Further Mathematics B (MEI)
Y414 Numerical methods

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
U1	Mark for correct units
G1	Mark for a correct feature on a graph
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1	(i)		$\frac{0.005}{4.145} = 0.00120(6272)$	B1	1.1		rounded to 3 s.f. or more for all three values
			$\frac{0.5}{341.5} = 0.00146(4128)$	B1	1.1		
				[2]			
	(ii)		$0.0012(06272) + 0.00146(4128)$	M1	1.1a	or $\dfrac{\frac{342}{4.15} - \frac{342.5}{4.145}}{\frac{342.5}{4.145}}$ oe	
			$= 0.00267(0401)$	A1	1.1		NB alternative approach gives – 0.002 66… and 0.002 67…
				[2]			

Question			Answer	Marks	AOs	Guidance	
2	**(i)**		2 -1 1 $\quad a$ $a-1 \quad 12-3a$ $a \quad 12-2a$ $11-a \quad b+3a-34$ 1 1 $\quad b+a-22$ $b-11$ b	**M1** **A1** **[2]**	**1.1** **1.1**	attempt at all three columns all correct	
2	**(ii)**		*their* $12-3a=0$ $a=4$ $b=22$	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	or *their* $12-2a=$ *their* a	
2	**(iii)**		$2+(x-0)(-1)+\frac{(x-0)(x-1)}{2!}(4)$ oe $[g(x)=]\ 2x^2-3x+2$	**M1** **A2** **[3]**	**1.1** **1.1** **1.1**	**A1** for two out of three terms correct	If **M0**, **B1** for each correct value.

Question			Answer	Marks	AOs	Guidance	
3	**(i)**		$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^4 - 1.5x^{\frac{1}{2}}$	**B1**	**1.1**		
			$x_{r+1} = x_r - \frac{x_r^5 - 3\sqrt{x_r} + 1}{\text{their f}'(x_r)}$	**M1**	**1.1**		
			$x_{r+1} = x_r - \frac{x_r^5 - 3\sqrt{x_r} + 1}{5x_r^4 - 1.5x_r^{-\frac{1}{2}}}$ oe	**A1**	**1.1**	all correct including subscripts	
				[3]			
3	**(ii)**	(*A*)	0.285 714 -0.090 086 -0.018 424 -0.000 707 0 -0.315 301 0.204 5186 0.038 372 4 0	**B1**	**1.1**	differences rounded to 3 s.f. or more	
				B1	**1.1**	ratios rounded to 3 s.f. or better	
				[2]			
3	**(ii)**	(*B*)	Ratios decreasing, so convergence faster than 1st order	**E1** **[1]**	**2.2a**		
4	**(i)**		0.99 × 10 000 000	**M1**	**3.4**		
			= $9 900 000	**A1** **[2]**	**1.1**	allow 990 000 000 cents	
4	**(ii)**		number of surplus cents is unlikely to be 99 in every case	**E1** **[1]**	**3.5b**		
4	**(iii)**		$0.495 \times 10\,000\,000 = \$4\,950\,000$	**B1** **[1]**	**3.4**	allow 4 95 000 000 cents	
4	**(iv)**		Assuming the number of surplus cents is equally likely to be any number from 0 to 99	**E1** **[1]**	**3.3**		

Question			Answer	Marks	AOs	Guidance	
5	(i)		it evaluates $\frac{1}{2^9}$	B1 [1]	2.4		
5	(ii)		=SUM(B2:B13) oe	M1 A1 [2]	1.2 2.5	use of SUM all correct	brackets optional
5	(iii)		The numbers from $\frac{1}{13^9}$ onwards are very small Adding them to the sum of the first 12 terms doesn't affect the 9th decimal place due to the limit of the accuracy of the spreadsheet	E1 E1 [2]	2.4 2.2a		
6	(i)	(*A*)	Sketch of $y = 2^x$ correct in both quadrants Sketch of $y = 3x + 2$ with one intercept identified	B1 B1 [2]	1.1 1.1		
6	(i)	(*B*)	2 real roots.	B1 [1]	2.2a		
6	(ii)		$x_1 = \frac{(2^0 - 2)}{3}, x_2 = \frac{(2^{x_1} - 2)}{3}$, etc − 0.416 994 42, − 0.417 005 545, − 0.417 007 471 − 0.41701 cao	M1 A1 A1 [3]	1.1 1.1 1.1	correct application of iterative formula at least twice convincing evidence of convergence	May be implied by 0, − 0.333 333…, − 0.420 99…,
6	(iii)		The second bracket should go in between 2 and / There should be * in between 3 and A	B1 B1 [2]	2.3 1.2	Accept other possible solutions Ignore any comment about the 10 in LOG10	

Question			Answer	Marks	AOs	Guidance	
7	(i)		$\sqrt{\frac{4.075}{5}}$ or $\sqrt{\frac{4.025}{5}}$	**M1**	**3.4**		
			$0.89721\ldots < b < 0.90277\ldots$	**A1**	**3.3**	Accept 3 d.p. or better, or exact values	
				[2]			
7	(ii)		$\frac{5\times0.9^{\sqrt{4+h}}-5\times0.9^{2}}{h}$	**M1**	**3.4**	with h = 0.01 or 0.001	
			− 0.1066..	**A1**	**1.1**		
			− 0.1067	**A1**	**1.1**		
				[3]			
7	(iii)		− 0.107 to 3 d.p.	**A1** **[1]**	**2.2a**		
7	(iv)		approximately 42 hours from the start.	**B1** **[1]**	**2.2a**	Or after a further 38 hours if clear	NB $4.05 \div 0.107 \approx 37.9$
7	(v)		e.g. the values of b, A at t = 4 and $\frac{dA}{dt}$ at t = 4 are only known to relatively low precision therefore only 1 s.f. is certain, although 2 s.f. is quite possible.	**E1** **[1]**	**3.2b**	this must include a sensible argument for 1 or 2 s.f., but no calculation necessary.	e.g. $\frac{dA}{dt}$ at t = 4 is small and even a minor change in its value might alter the final result significantly

Question			Answer	Marks	AOs	Guidance	
8	(i)	(*A*)	$M_1 = 1 \times \sqrt{(1 - 0.5^3)} = 0.935414$ $T_1 = \frac{1}{2} \times \left\{\sqrt{(1 - 0^3)} + \sqrt{(1 - 1^3)}\right\} = 0.5$	M1 A1 M1 A1 [4]	1.1a 1.1 1.1a 1.1		
8	(i)	(*B*)	$[S_2 =]\frac{(2 \times 0.935414 + 0.5)}{3} = 0.790276$	M1 A1 [2]	2.1 1.1	Reasoning must be shown Follow through their M_1 and T_1 .	
8	(ii)	(*A*)	0.841	B1 [1]	3.1a		
8	(ii)	(*B*)	The 2 best estimates from each rule agree to 3 d.p.	B1 [1]	2.4		
8	(iii)		e.g. $=(4*C9-C8)/3$ e.g. $=(2*B8+C8)/3$	M1 A1 M1 A1 [4]	1.1 1.1 1.1 1.1	condone omission of * or = condone omission of * or =	or clear evidence of use of a correct formula or clear evidence of use of a correct formula
8	(iv)		e.g.The evidence shows that Simpson's rule is not a fourth order method in this case extrapolation using *r* = 0.353 is more reliable	M1 A1 [2]	2.2b 3.1a	accept other suitable comment	
8	(v)		0.841 274 922 is best Simpson's estimate from table (cell D9) 0.841 3 is certain; The estimate from the table and the extrapolated value agree to 4 decimal places.	M1 A1 [2]	3.1a 2.2b	by comparison of extrapolated value with best Simpson's value	allow 0.841 31 or 0.841 309 since extrapolation results in significant improvement NB it is actually accurate to 6 dp, but there is insufficient evidence to state this

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1i	2	0	0	0	2
1ii	2	0	0	0	2
2i	2	0	0	0	2
2ii	3	0	0	0	3
2iii	3	0	0	0	3
3i	3	0	0	0	3
3iiA	2	0	0	0	2
3iiB	0	1	0	0	1
4i	1	0	0	1	2
4ii	0	0	0	1	1
4iii	0	0	0	1	1
4iv	0	0	0	1	1
5i	0	1	0	0	1
5ii	1	1	0	0	2
5iii	0	2	0	0	2
6iA	2	0	0	0	2
6iB	0	1	0	0	1
6ii	3	0	0	0	3
6iii	1	1	0	0	2
7i	0	0	0	2	2
7ii	2	0	0	1	3
7iii	0	1	0	0	1
7iv	0	1	0	0	1
7v	0	0	1	0	1
8iA	4	0	0	0	4
8iB	1	1	0	0	2
8iiA	0	0	1	0	1
8iiB	0	1	0	0	1
8iii	4	0	0	0	4
8iv	0	1	1	0	2
8v	0	1	1	0	2
Totals	36	13	4	7	60

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AS Level Further Mathematics B (MEI)

Y414 Numerical Methods

Printed Answer Booklet

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

OCR supplied materials:
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You must have:
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First name									
Last name									
Centre number						**Candidate number**			

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1(i)	
1(ii)	
2 (i)	

2 (ii)	
2 (iii)	

DO NOT WRITE IN THIS SPACE

3 (i)	
3 (ii) (*A*)	
3 (ii) (*B*)	

4 (i)	
4 (ii)	
4(iii)	
4 (iv)	
5 (i)	
5 (ii)	
5 (iii)	

6 (i) (*A*)	
6 (i) (*B*)	

6 (ii)	
6 (iii)	

DO NOT WRITE IN THIS SPACE

7 (i)	
7 (ii)	
7 (iii)	
7 (iv)	
7 (v)	

8 (i) (*A*)	
8 (i) (*B*)	

8 (ii) (*A*)	
8 (ii) (*B*)	
8 (iii)	
8 (iv)	
8 (v)	

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