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AS Level Further Mathematics B (MEI)
Y412 Statistics a
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions

1 The number of failures of a machine each week at a factory is modelled by a Poisson distribution with mean 0.45.

(i) Write down the variance of the distribution. **[1]**

(ii) Find the probability that there are exactly 2 failures in a week. **[1]**

(iii) State a distribution which can be used to model the number of failures in a period of 4 weeks. **[2]**

(iv) Find the probability that there are at least 2 failures in a period of 4 weeks. **[2]**

2 The discrete random variable Y is uniformly distributed over the values $\{12, 13, \ldots, 20\}$.

(i) Write down $\mathrm{P}(Y < 15)$. **[1]**

(ii) Two independent observations of Y are taken. Find the probability that one of these values is less than 15 and the other is greater than 15. **[3]**

(iii) Find $\mathrm{P}(Y > \mathrm{E}(Y))$. **[2]**

3 **In this question you must show detailed reasoning.**

A student is investigating what people think about organic food. She wishes to see if there is any difference between the opinions of females and males. She takes a random sample of 100 people and asks each of them if they think that organic food is better for their health than non-organic food. She will use the data to conduct a hypothesis test. The table below shows the opinions of these 100 people.

		Sex	
		Female	Male
Opinion on organic food	Organic better	35	18
	Not better	22	25

(i) Explain why the student should use a random sample. **[2]**

(ii) Carry out a test at the 5% significance level to examine whether there is any association between a person's sex and their opinion on organic food. Show your calculations. **[8]**

4 The discrete random variable X has probability distribution defined by

$$\mathrm{P}(X = r) = k(2r - 1) \qquad \text{for } r = 1, 2, 3, 4, 5, 6, \text{ where } k \text{ is a constant.}$$

(i) Complete the table in the Printed Answer Booklet giving the probabilities in terms of k.

r	1	2	3	4	5	6
$\mathrm{P}(X = r)$						

[1]

(ii) Show that the value of k is $\frac{1}{36}$. **[2]**

(iii) Draw a graph to illustrate the distribution. **[2]**

(iv) In this question you must show detailed reasoning.

Find

- $\mathrm{E}(X)$
- $\mathrm{Var}(X)$. **[5]**

A game consists of a player throwing two fair dice. The score is the maximum of the two values showing on the dice.

(v) Show that the probability of a score of 3 is $\frac{5}{36}$. **[2]**

(vi) Show that the probability distribution for the score in the game is the same as the probability distribution of the random variable X. **[3]**

(vii) The game is played three times.

Find

- the mean of the total of the three scores.
- the variance of the total of the three scores. **[3]**

5 In a recent report, it was stated that 40% of working people have a degree. For the whole of this question, you should assume that this is true.

A researcher wishes to interview a working person who has a degree. He asks working people at random whether they have a degree and counts the number of people he has to ask until he finds one with a degree.

(i) Find the probability that he has to ask 5 people. **[2]**

(ii) Find the mean number of people the researcher has to ask. **[1]**

Subsequently, the researcher decides to take a random sample from the population of working people.

(iii) A random sample of 5 working people is chosen. What is the probability that at least one of them has a degree? **[2]**

(iv) How large a random sample of working people would the researcher need to take to ensure that the probability that at least one person has a degree is 0.99 or more? **[3]**

6 A motorist decides to check the fuel consumption, y miles per gallon, of her car at particular speeds, x mph, on flat roads. She carries out the check on a suitable stretch of motorway. Fig.6 shows her results.

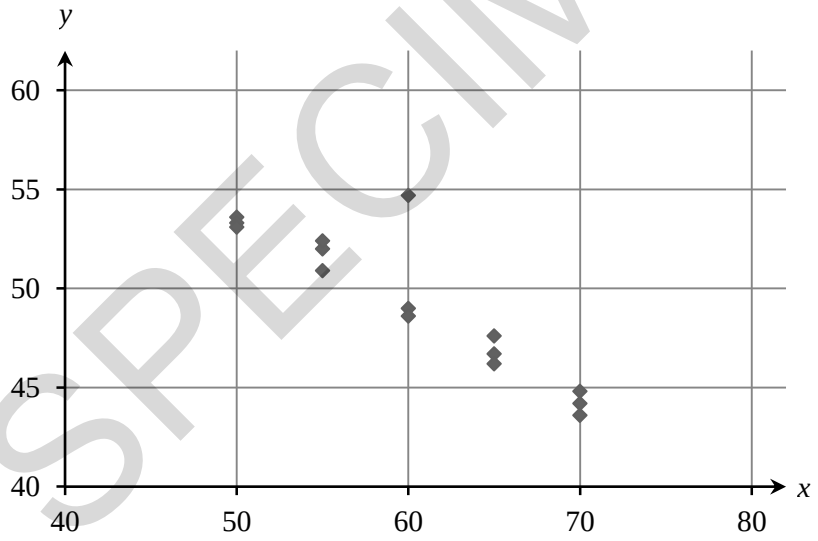

Fig.6

(i) Explain why it would not be appropriate to carry out a hypothesis test for correlation based on the product moment correlation coefficient. **[2]**

(ii) (*A*) One of the results is an outlier. Circle the outlier on the copy of Fig. 6 in the Printed Answer Booklet. **[1]**

(*B*) Suggest one possible reason for the outlier in part **(ii)** (*A*) not being used in any analysis. **[1]**

The motorist decides to remove this item of data from any analysis. The table below shows part of a spreadsheet that was used to analyse the 14 remaining data items (with the outlier removed). Some rows of the spreadsheet have been deliberately omitted.

	Data item	x	y	x^2	y^2	xy
	1	50	53.6	2500	2872.96	2680
	2	50	53.3	2500	2840.89	2665
	13	70	44.8	4900	2007.04	3136
	14	70	44.2	4900	1953.64	3094
Sum		840	686	51150	33779.7	40812

(iii) Calculate the equation of the regression line of y on x. **[4]**

(iv) Use the equation of the regression line to predict the fuel consumption of the car at

(*A*) 58 mph,

(*B*) 30 mph. **[2]**

(v) Comment on the reliability of your predictions in part **(iv)**. **[2]**

END OF QUESTION PAPER

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OCR
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…day June 20XX – Morning/Afternoon

AS Level Further Mathematics B (MEI)

Y412 Statistics a

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 16 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
U1	Mark for correct units
G1	Mark for a correct feature on a graph
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

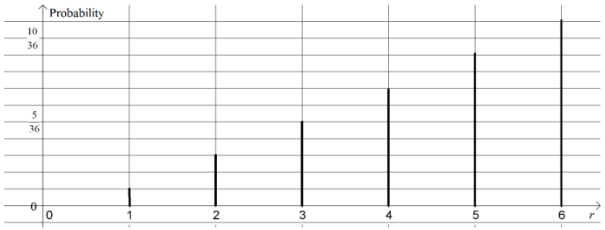
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1	(i)		Variance = 0.45	B1 [1]	1.2		
	(ii)		P(2 failures) = 0.0646	B1 [1]	1.1	BC	
	(iii)		Poisson Parameter = 1.8	B1 B1 [2]	3.3 1.1a		
	(iv)		Using $\lambda = 1.8$ P(at least 2 in 4 weeks) = 1 – 0.4628 = 0.5372	M1 A1 [2]	1.1a 1.1	For 0.4628 OR 1 - P($Y \leq 1$) BC cao	
2	(i)		$P(Y < 15) = \frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1}{3}$	B1 [1]	1.1		
	(ii)		$\frac{1}{3}\times\frac{5}{9}+\frac{5}{9}\times\frac{1}{3}$ $= \frac{10}{27}$	B1 M1 A1 [3]	1.1 3.1a 1.1	For $\frac{5}{9}$ For sum of two products of fractions FT from (i) if all probabilities in (0, 1)	
	(iii)		E(Y) =16 $P(Y>16)=\frac{4}{9}$	B1 B1 [2]	1.1 1.1	soi FT their E(Y) if in [15,17]	

Question			Answer	Marks	AOs	Guidance	
3	(i)		A random sample enables proper inference about the population to be undertaken.	**B2** **[2]**	**2.4** **2.4**	**B2** for correct explanation as shown in answer column OR **B1** for partially correct explanation e.g. a random sample is less likely to be biased	

Question			Answer	Marks	AOs	Guidance	
3	(ii)		DR H_0: no association between sex and opinion on health benefits of organic food H_1: some association between sex and opinion on health benefits of organic food	**B1**	**3.3**	Allow hypotheses and conclusion in terms of independence.	
			<table><tr><td>Expected frequency</td><td>Female</td><td>Male</td></tr><tr><td>Organic better</td><td>30.21</td><td>22.79</td></tr><tr><td>Not better</td><td>26.79</td><td>20.21</td></tr></table>	**M1** **A1**	**3.4** **1.1**	For expected frequencies	
			<table><tr><td>Contribution</td><td>Female</td><td>Male</td></tr><tr><td>Organic better</td><td>0.7595</td><td>1.0068</td></tr><tr><td>Not better</td><td>0.8564</td><td>1.1353</td></tr></table>	**B1**	**1.1**	Expected NB This **B1** mark cannot be implied by a correct final value of χ^2	
			$\chi^2 = 3.76$	**B1**	**1.1**	Yates correction not expected – if used, $\chi^2 = 3.01$.	
			Refer to χ_1^2 Critical value at 5% level = 3.84 3.76 < 3.84 Result is not significant	**B1** **B1**	**2.5** **2.2b**	Degrees of freedom = 1	
			There is not enough evidence to suggest that there is association between sex and opinion on health benefits of organic food.	**E1** **[8]**	**3.5a**	NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first **B1** and do not award final **E1**	

Question			Answer	Marks	AOs	Guidance	
4	(i)		r | 1 | 2 | 3 | 4 | 5 | 6 $P(X = r)$ | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$	B1 [1]	1.1		
4	(ii)		$k + 3k + 5k + 7k + 9k + 11k = 1$ $36k = 1$ so $k = \frac{1}{36}$ so $k = \frac{1}{36}$ AG	M1 A1 [2]	2.4 1.1	For equation	
4	(iii)		 [2]	B1 B1 [2]	1.1 1.1	For heights For axes and labels	

Question			Answer	Marks	AOs	Guidance	
4	(iv)		DR $\mathrm{E}(X)=1\times\frac{1}{36}+\ 2\times\frac{2}{36}+\ 3\times\frac{5}{36}+4\times\frac{7}{36}+5\times\frac{9}{36}$ $+6\times\frac{11}{36}$ $=\frac{161}{36}\quad\left[=4.47\right]$ $\mathrm{E}\left(X^2\right)=1^2\times\frac{1}{36}+2^2\times\frac{3}{36}+3^2\times\frac{5}{36}+4^2\times\frac{7}{36}+5^2\times\frac{9}{36}$ $+6^2\times\frac{11}{36}$ $=\frac{791}{36}$ $\mathrm{Var}\left(X\right)\ =\frac{791}{36}-\left(\frac{161}{36}\right)^2=\frac{2555}{1296}\quad\left(=\ 1.97\right)$	**M1** **A1** **M1** **M1** **A1** **[5]**	**1.1a** **1.1** **1.1** **1.2** **1.1**		
4	(v)		Combinations which lead to score of 3 are (1, 3), (2, 3), (3, 3), (3, 2), (3, 1) 36 possible outcomes so Probability $=\frac{5}{36}$ AG	**M1** **A1** **[2]**	**2.1** **1.1**	May be in a sample space	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(vi)</td><td></td><td>Finding probability distribution for score
<table><tr><td>Score</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>2</td><td>2</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>5</td><td>6</td></tr><tr><td>5</td><td>5</td><td>5</td><td>5</td><td>5</td><td>5</td><td>6</td></tr><tr><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td></tr></table>
Probabilities are consecutive odd numbers out of 36</td><td>B1

B1

E1

[3]</td><td>1.1

2.1

2.2a</td><td>Strategy e.g. use of sample space

Working through strategy
e.g. completing sample space

Matching probabilities for score with those for X. Must be convincing</td><td></td></tr>
<tr><td>4</td><td>(vii)</td><td></td><td>Total $= X_1 + X_2 + X_3$
$\mathrm{E}(X_1 + X_2 + X_3) = \frac{161}{36} + \frac{161}{36} + \frac{161}{36} = \frac{483}{36}$ $(=13.42)$
$\mathrm{Var}(X_1 + X_2 + X_3) = \frac{2555}{1296} + \frac{2555}{1296} + \frac{2555}{1296} = 5.91$</td><td>B1
B1

B1

[3]</td><td>1.1a
1.1

1.1</td><td>
FT from (iv)

FT from (iv)</td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
5	(i)		$P(5) = 0.6^4 \times 0.4$ $= 0.05184$	M1 A1 [2]	3.3 1.1		
5	(ii)		Mean = 2.5	B1 [1]	3.4		
5	(iii)		$[1 -]\ 0.6^5$ 0.9222	M1 A1 [2]	1.1a 1.1		
5	(iv)		$1 - 0.6^n = 0.99$ $n = \frac{\log 0.01}{\log 0.6} [= 9.015...]$ 10 [people]	M1 M1 A1 [3]	3.1b 1.1 3.2a		

Question			Answer	Marks	AOs	Guidance	
6	(i)		Because this test is only valid for random on random data … and the speeds are controlled.	E1 E1 [2]	3.5a 3.5b		
6	(ii)	(*A*)	(60, 54.7) is an outlier.	B1 [1]	1.2		
6	(ii)	(*B*)	Because it is not representative	E1 [1]	2.4	E.g. differs markedly from other values at same speed OR could be an error	
6	(iii)		$\overline{x} = 60,\ \overline{y} = 49$ $b = \frac{S_{xy}}{S_{xx}} = \frac{40812 - 840 \times 686/14}{51150 - 840^2/14} = \frac{-348}{750}$ OR $b = \frac{40812/14 - 60 \times 49}{51150/14 - 49^2} = \frac{-24.857}{53.571}$ $= -0.464$ hence least squares regression line is: $\Rightarrow \quad y - 49 = -0.464\,(x - 60)$ $\Rightarrow \quad y = -0.464x + 76.84$	 M1 A1 M1 A1 [4]	 1.1a 1.1 3.3 1.1	 For attempt at gradient (*b*) For –0.464 For equation of line FT for complete equation	
6	(iv)	*(A)* *(B)*	49.9 62.9	B1 B1 [2]	3.4 3.4		
6	(v)		First answer is reliable as interpolation and the points lie close to the line Second answer less likely to be reliable as extrapolation	E1 E1 [2]	3.5a 3.5b		

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	1	0	0	0	1
1(ii)	1	0	0	0	1
1(iii)	1	0	0	1	2
1(iv)	2	0	0	0	2
2(i)	1	0	0	0	1
2(ii)	2	0	1	0	3
2(iii)	2	0	0	0	2
3(i)	0	2	0	0	2
3(ii)	3	2	0	3	8
4(i)	1	0	0	0	1
4(ii)	1	1	0	0	2
4(iii)	2	0	0	0	2
4(iv)	5	0	0	0	5
4(v)	1	1	0	0	2
4(vi)	1	2	0	0	3
4(vii)	3	0	0	0	3
5(i)	1	0	0	1	2
5(ii)	0	0	0	1	1
5(iii)	2	0	0	0	2
5(iv)	1	0	2	0	3
6(i)	0	0	0	2	2
6(ii)A	1	0	0	0	1
6(ii)B	0	1	0	0	1
6(iii)	3	0	0	1	4
6(iv)	0	0	0	2	2
6(v)	0	0	0	2	2
Totals	35	9	3	13	60

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First name	
Last name	
Centre number	**Candidate number**

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1 (i)	
1 (ii)	
1 (iii)	
1 (iv)	
2 (i)	
2 (ii)	

2 (iii)	
3 (i)	
3 (ii)	

(answer space continued on next page)

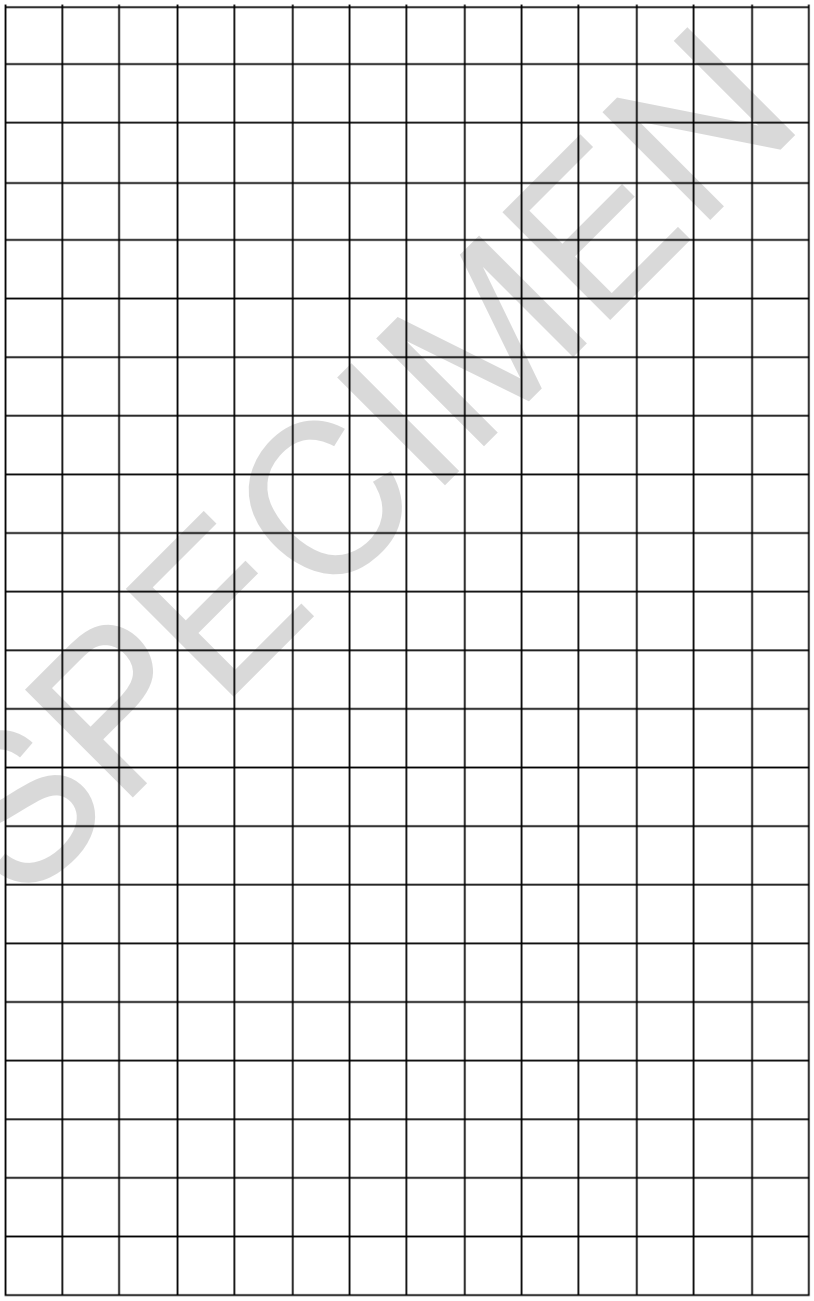
3 (ii)	**(continued)**

4 (i)

r	1	2	3	4	5	6
$P(X = r)$						

4 (ii)

4 (iii)



4 (iv)

4 (v)

4 (vi)

4 (vii)

5 (i)

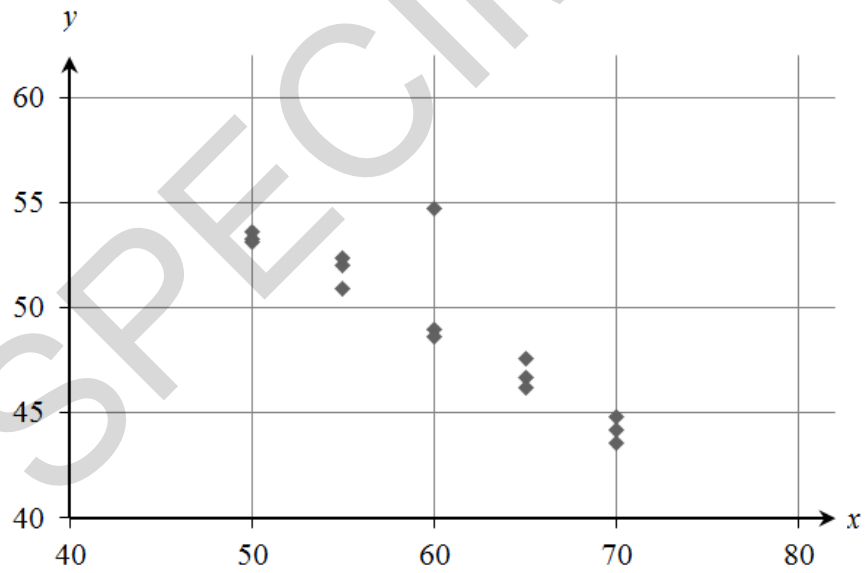
5 (ii)

5 (iii)	
5 (iv)	

DO NOT WRITE IN THIS SPACE

6 (i)

6 (ii) (*A*)



6 (ii) (*B*)

6 (iii)

6 (iv) (*A*)	
6 (iv) (*B*)	
6 (v)	