OCR
Oxford Cambridge and RSA

Advanced Subsidiary GCE (H635)
Advanced GCE (H645)

Further Mathematics B (MEI)

Formulae Booklet

3369126002

AS and A Level Further Mathematics B (MEI)

The information in this booklet is for the use of candidates following the Advanced Subsidiary in Further Mathematics B (MEI)(H635) or the Advanced GCE in Further Mathematics B (MEI) (H645) course.

The formulae booklet will be printed for distribution with the examination papers.

Copies of this booklet may be used for teaching.

This document consists of **16** pages.

Instructions to Exams Officer/Invigilator

- **Do not send this Data Sheet for marking; it should be retained in the centre or destroyed.**

Contents

A level Mathematics

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}, \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small Angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0 : x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Sample Variance

$s^2 = \frac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

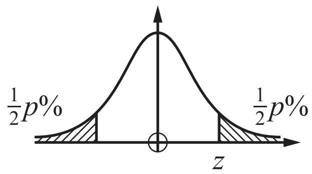
If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$
Mean of X is np

Hypothesis test for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two and three dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

-

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Core Pure

Complex Numbers

De Moivre's theorem:
$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

Roots of unity:

The roots of $z^n = 1$ are given by $z = \exp\left(\frac{2\pi k}{n}\mathrm{i}\right)$ for $k = 0, 1, 2, \ldots, n-1$

Vectors and 3-D geometry

Cartesian equation of a plane is
$n_1 x + n_2 y + n_3 z + d = 0$

Cartesian equation of a line in 3-D is
$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$

Vector product $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$ where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.

Distance between skew lines is $\left|\frac{\mathbf{d}_1 \times \mathbf{d}_2}{|\mathbf{d}_1 \times \mathbf{d}_2|}.(\mathbf{a}_1 - \mathbf{a}_2)\right|$ where $\mathbf{a}_1$ is the position vector of a point on the first line and $\mathbf{d}_1$ is parallel to the first line, similarly for the second line.

Distance between point (x_1, y_1) and line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between point (x_1, y_1, z_1) and plane $n_1 x + n_2 y + n_3 z + d = 0$ is $\frac{|n_1 x_1 + n_2 y_1 + n_3 z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2 + 1)}]$

$\operatorname{arcosh} x = \ln[x + \sqrt{(x^2 - 1)}],\ x \geq 1$

$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$

Calculus

$f(x)$	$f'(x)$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$

$f(x)$	$\int f(x)\,dx$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2}) \quad (x > a)$

The mean value of $f(x)$ on the interval $[a,b]$ is $\dfrac{1}{b-a}\int_a^b f(x)\,dx$

Area of sector enclosed by polar curve is $\dfrac{1}{2}\int r^2\,d\theta$

Series

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \quad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (\lvert x\rvert < 1, n \in \mathbb{R})$$

Mechanics

Motion in a circle

For motion in a circle,

tangential velocity is $v = r\dot{\theta}$

radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

tangential acceleration is $r\ddot{\theta}$

Further Pure with Technology

Numerical solution of differential equations

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method: $x_{n+1} = x_n + h \quad y_{n+1} = y_n + h\mathrm{f}(x_n, y_n)$

Modified Euler method (A Runge-Kutta method of order 2):

$k_1 = h\mathrm{f}(x_n, y_n)$

$k_2 = h\mathrm{f}(x_n + h, y_n + k_1)$

$x_{n+1} = x_n + h, \ y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$

Runge-Kutta method of order 4:

$k_1 = h\mathrm{f}(x_n, y_n)$

$k_2 = h\mathrm{f}(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = h\mathrm{f}(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = h\mathrm{f}(x_n + h, y_n + k_3)$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Gradient of tangent to a polar curve

For a curve $r = \mathrm{f}(\theta)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta}$

Extra Pure

Multivariable calculus

$\nabla \mathrm{g} = \mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{g}}{\partial x} \\ \frac{\partial \mathrm{g}}{\partial y} \\ \frac{\partial \mathrm{g}}{\partial z} \end{pmatrix}$. If $\mathrm{g}(x, y, z)$ can be written as $z = \mathrm{f}(x, y)$ then $\mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{f}}{\partial x} \\ \frac{\partial \mathrm{f}}{\partial y} \\ -1 \end{pmatrix}$

Numerical methods

Solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

For the iteration $x_{n+1} = \mathrm{g}(x_n)$ the relaxed iteration is $x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n)$.

Numerical integration

To estimate $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$:

The midpoint rule:

$M_n = h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \ldots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$ where $h = \dfrac{b-a}{n}$

The trapezium rule:

$T_n = \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$ where $h = \dfrac{b-a}{n}$

Simpson's rule

$$S_{2n} = \frac{1}{3}h\{(y_0 + y_{2n}) + 4(y_1 + y_3 + \ldots + y_{2n-1}) + 2(y_2 + y_4 + \ldots + y_{2n-2})\}$$

where $h = \dfrac{b-a}{2n}$

These are related as follows:

$$T_{2n} = \frac{1}{2}(M_n + T_n)$$

$$S_{2n} = \frac{1}{3}(2M_n + T_n) = \frac{1}{3}(4T_{2n} - T_n)$$

Interpolation

Newton's forward difference interpolation formula:

$$\mathrm{f}(x) = \mathrm{f}(x_0) + \frac{(x-x_0)}{h}\Delta \mathrm{f}(x_0) + \frac{(x-x_0)(x-x_1)}{2!h^2}\Delta^2 \mathrm{f}(x_0) + \ldots$$

Lagrange's polynomial:

$$\mathrm{P}_n(x) = \sum \mathrm{L}_r(x)\mathrm{f}(x_r) \quad \text{where} \quad \mathrm{L}_r(x) = \prod_{\substack{i=0 \\ i \neq r}}^{n} \frac{x - x_i}{x_r - x_i}$$

Statistics

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $\mathrm{P}(X = x_i) = p_i$

Expectation: $\mu = \mathrm{E}(X) = \sum x_i p_i$

Variance: $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	Probability	E(X)	Var(X)
Uniform distribution over 1, 2, …, n	$P(X = r) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2 - 1)$
Geometric distribution	$P(X = r) = q^{r-1}p$ $q = 1 - p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson distribution	$P(X = r) = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}$		

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n},\ S_{yy} = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},\ S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

product moment correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

least squares regression line of y on x is $y - \bar{y} = b(x - \bar{x})$ where $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}$

least squares regression line of x on y is $x - \bar{x} = b'(y - \bar{y})$ where $b' = \frac{S_{xy}}{S_{yy}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}}$

Spearman's coefficient of rank correlation:

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$$

Confidence intervals

To calculate a confidence interval for a mean or difference in mean in different circumstances, use the given distribution to calculate the critical value, k.

To estimate…	Confidence interval	Distribution
a mean	$\bar{x} \pm k\frac{\sigma}{\sqrt{n}}$	N(0, 1)
a mean	$\bar{x} \pm k\frac{s}{\sqrt{n}}$	t_{n-1}
difference in mean of paired populations	treat differences as a single distribution	

Hypothesis tests

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$	
Spearman's rank correlation test	$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$	
χ^2 test	$\sum \frac{(f_o - f_e)^2}{f_e}$	χ_v^2
Normal test for a mean	$\frac{\bar{x}-\mu}{\left(\sigma/\sqrt{n}\right)}$	N(0, 1)
t-test for a mean	$\frac{\bar{x}-\mu}{\left(s/\sqrt{n}\right)}$	t_{n-1}
Wilcoxon single sample test	A statistic T is calculated from the ranked data	

Continuous distributions

X is a continuous random variable with probability density function (pdf) $\mathrm{f}(x)$

Expectation: $\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$

Variance: $\sigma^2 = \mathrm{Var}(X) = \int (x-\mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$

Cumulative distribution function $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$

	E(X)	Var(X)
Continuous uniform distribution over $[a,b]$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$

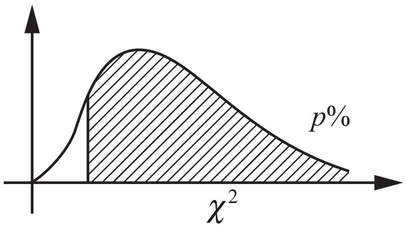
Critical values for the product moment correlation coefficient, r

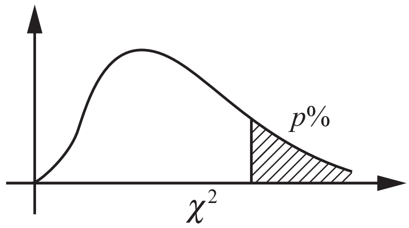
1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					n				
1	-	-	-	-	31	0.3009	0.3550	0.4158	0.4556
2	-	-	-	-	32	0.2960	0.3494	0.4093	0.4487
3	0.9877	0.9969	0.9995	0.9999	33	0.2913	0.3440	0.4032	0.4421
4	0.9000	0.9500	0.9800	0.9900	34	0.2869	0.3388	0.3972	0.4357
5	0.8054	0.8783	0.9343	0.9587	35	0.2826	0.3338	0.3916	0.4296
6	0.7293	0.8114	0.8822	0.9172	36	0.2785	0.3291	0.3862	0.4238
7	0.6694	0.7545	0.8329	0.8745	37	0.2746	0.3246	0.3810	0.4182
8	0.6215	0.7067	0.7887	0.8343	38	0.2709	0.3202	0.3760	0.4128
9	0.5822	0.6664	0.7498	0.7977	39	0.2673	0.3160	0.3712	0.4076
10	0.5494	0.6319	0.7155	0.7646	40	0.2638	0.3120	0.3665	0.4026
11	0.5214	0.6021	0.6851	0.7348	41	0.2605	0.3081	0.3621	0.3978
12	0.4973	0.5760	0.6581	0.7079	42	0.2573	0.3044	0.3578	0.3932
13	0.4762	0.5529	0.6339	0.6835	43	0.2542	0.3008	0.3536	0.3887
14	0.4575	0.5324	0.6120	0.6614	44	0.2512	0.2973	0.3496	0.3843
15	0.4409	0.5140	0.5923	0.6411	45	0.2483	0.2940	0.3457	0.3801
16	0.4259	0.4973	0.5742	0.6226	46	0.2455	0.2907	0.3420	0.3761
17	0.4124	0.4821	0.5577	0.6055	47	0.2429	0.2876	0.3384	0.3721
18	0.4000	0.4683	0.5425	0.5897	48	0.2403	0.2845	0.3348	0.3683
19	0.3887	0.4555	0.5285	0.5751	49	0.2377	0.2816	0.3314	0.3646
20	0.3783	0.4438	0.5155	0.5614	50	0.2353	0.2787	0.3281	0.3610
21	0.3687	0.4329	0.5034	0.5487	51	0.2329	0.2759	0.3249	0.3575
22	0.3598	0.4227	0.4921	0.5368	52	0.2306	0.2732	0.3218	0.3542
23	0.3515	0.4132	0.4815	0.5256	53	0.2284	0.2706	0.3188	0.3509
24	0.3438	0.4044	0.4716	0.5151	54	0.2262	0.2681	0.3158	0.3477
25	0.3365	0.3961	0.4622	0.5052	55	0.2241	0.2656	0.3129	0.3445
26	0.3297	0.3882	0.4534	0.4958	56	0.2221	0.2632	0.3102	0.3415
27	0.3233	0.3809	0.4451	0.4869	57	0.2201	0.2609	0.3074	0.3385
28	0.3172	0.3739	0.4372	0.4785	58	0.2181	0.2586	0.3048	0.3357
29	0.3115	0.3673	0.4297	0.4705	59	0.2162	0.2564	0.3022	0.3328
30	0.3061	0.3610	0.4226	0.4629	60	0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					n				
1	-	-	-	-	31	0.3012	0.3560	0.4185	0.4593
2	-	-	-	-	32	0.2962	0.3504	0.4117	0.4523
3	-	-	-	-	33	0.2914	0.3449	0.4054	0.4455
4	1.0000	-	-	-	34	0.2871	0.3396	0.3995	0.4390
5	0.9000	1.0000	1.0000	-	35	0.2829	0.3347	0.3936	0.4328
6	0.8286	0.8857	0.9429	1.0000	36	0.2788	0.3300	0.3882	0.4268
7	0.7143	0.7857	0.8929	0.9286	37	0.2748	0.3253	0.3829	0.4211
8	0.6429	0.7381	0.8333	0.8810	38	0.2710	0.3209	0.3778	0.4155
9	0.6000	0.7000	0.7833	0.8333	39	0.2674	0.3168	0.3729	0.4103
10	0.5636	0.6485	0.7455	0.7939	40	0.2640	0.3128	0.3681	0.4051
11	0.5364	0.6182	0.7091	0.7545	41	0.2606	0.3087	0.3636	0.4002
12	0.5035	0.5874	0.6783	0.7273	42	0.2574	0.3051	0.3594	0.3955
13	0.4835	0.5604	0.6484	0.7033	43	0.2543	0.3014	0.3550	0.3908
14	0.4637	0.5385	0.6264	0.6791	44	0.2513	0.2978	0.3511	0.3865
15	0.4464	0.5214	0.6036	0.6536	45	0.2484	0.2945	0.3470	0.3822
16	0.4294	0.5029	0.5824	0.6353	46	0.2456	0.2913	0.3433	0.3781
17	0.4142	0.4877	0.5662	0.6176	47	0.2429	0.2880	0.3396	0.3741
18	0.4014	0.4716	0.5501	0.5996	48	0.2403	0.2850	0.3361	0.3702
19	0.3912	0.4596	0.5351	0.5842	49	0.2378	0.2820	0.3326	0.3664
20	0.3805	0.4466	0.5218	0.5699	50	0.2353	0.2791	0.3293	0.3628
21	0.3701	0.4364	0.5091	0.5558	51	0.2329	0.2764	0.3260	0.3592
22	0.3608	0.4252	0.4975	0.5438	52	0.2307	0.2736	0.3228	0.3558
23	0.3528	0.4160	0.4862	0.5316	53	0.2284	0.2710	0.3198	0.3524
24	0.3443	0.4070	0.4757	0.5209	54	0.2262	0.2685	0.3168	0.3492
25	0.3369	0.3977	0.4662	0.5108	55	0.2242	0.2659	0.3139	0.3460
26	0.3306	0.3901	0.4571	0.5009	56	0.2221	0.2636	0.3111	0.3429
27	0.3242	0.3828	0.4487	0.4915	57	0.2201	0.2612	0.3083	0.3400
28	0.3180	0.3755	0.4401	0.4828	58	0.2181	0.2589	0.3057	0.3370
29	0.3118	0.3685	0.4325	0.4749	59	0.2162	0.2567	0.3030	0.3342
30	0.3063	0.3624	0.4251	0.4670	60	0.2144	0.2545	0.3005	0.3314

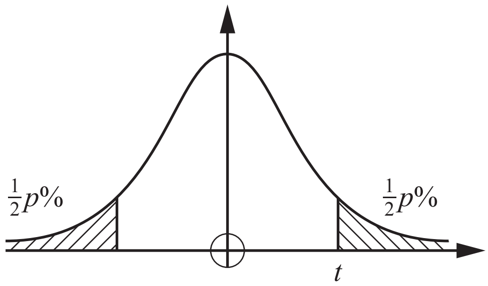
Percentage points of the χ^2 (chi-squared) distribution





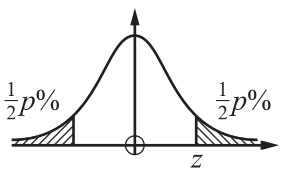
p%	99	97.5	95	90	10	5	2.5	1	0.5
$\nu = 1$	.0001	.0010	.0039	.0158	2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211	4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490	13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312	23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09	24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Percentage points of the *t* distribution



v \ $p\%$	10	5	2	1	
1	6.314	12.71	31.82	63.66	
2	2.920	4.303	6.965	9.925	
3	2.353	3.182	4.541	5.841	
4	2.132	2.776	3.747	4.604	
5	2.015	2.571	3.365	4.032	
6	1.943	2.447	3.143	3.707	
7	1.895	2.365	2.998	3.499	
8	1.860	2.306	2.896	3.355	
9	1.833	2.262	2.821	3.250	
10	1.812	2.228	2.764	3.169	
11	1.796	2.201	2.718	3.106	
12	1.782	2.179	2.681	3.055	
13	1.771	2.160	2.650	3.012	
14	1.761	2.145	2.624	2.977	
15	1.753	2.131	2.602	2.947	
20	1.725	2.086	2.528	2.845	
30	1.697	2.042	2.457	2.750	
50	1.676	2.009	2.403	2.678	
100	1.660	1.984	2.364	2.626	
∞	1.645	1.960	2.326	2.576	= percentage points of the Normal distribution N(0, 1)

Percentage points of the normal distribution



p	10	5	2	1
z	1.645	1.960	2.326	2.576

Critical values for the Wilcoxon Single Sample test

1-tail	5%	2½%	1%	½%	1-tail	5%	2½%	1%	½%
2-tail	10%	5%	2%	1%	2-tail	10%	5%	2%	1%
n					*n*				
					26	110	98	84	75
2	–	–	–	–	27	119	107	92	83
3	–	–	–	–	28	130	116	101	91
4	–	–	–	–	29	140	126	110	100
5	0	–	–	–	30	151	137	120	109
6	2	0	–	–	31	163	147	130	118
7	3	2	0	–	32	175	159	140	128
8	5	3	1	0	33	187	170	151	138
9	8	5	3	1	34	200	182	162	148
10	10	8	5	3	35	213	195	173	159
11	13	10	7	5	36	227	208	185	171
12	17	13	9	7	37	241	221	198	182
13	21	17	12	9	38	256	235	211	194
14	25	21	15	12	39	271	249	224	207
15	30	25	19	15	40	286	264	238	220
16	35	29	23	19	41	302	279	252	233
17	41	34	27	23	42	319	294	266	247
18	47	40	32	27	43	336	310	281	261
19	53	46	37	32	44	353	327	296	276
20	60	52	43	37	45	371	343	312	291
21	67	58	49	42	46	389	361	328	307
22	75	65	55	48	47	407	378	345	322
23	83	73	62	54	48	426	396	362	339
24	91	81	69	61	49	446	415	379	355
25	100	89	76	68	50	466	434	397	373

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