OCR
Oxford Cambridge and RSA

Advanced Subsidiary GCE (H235)

Further Mathematics A

Formulae Booklet

AS Level Further Mathematics A

The information in this booklet is for the use of candidates following the Advanced Subsidiary course.

The formulae booklet will be printed for distribution with the examination papers.

Copies of this booklet may be used for teaching.

This document consists of **8** pages.

Instructions to Exams Officer/Invigilator

- **Do not send this Formulae Booklet for marking; it should be retained in the centre or destroyed.**

Pure Mathematics

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1} b + {}^n\mathrm{C}_2 a^{n-2} b^2 + \ldots + {}^n\mathrm{C}_r\, a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

Matrix transformations

Reflection in the line $y = \pm x$: $\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Rotations through θ about the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Complex numbers

Circles: $|z-a| = k$

Half lines: $\arg(z-a) = \alpha$

Lines: $|z-a| = |z-b|$

Vectors and 3-D coordinate geometry

Cartesian equation of the line through the point A with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ in direction $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ is $\dfrac{x-a_1}{u_1} = \dfrac{y-a_2}{u_2} = \dfrac{z-a_3}{u_3} (= \lambda)$

Vector product: $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

Statistics

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \textbf{ or } \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $\mathrm{P}(X = x_i) = p_i$

Expectation: $\mu = \mathrm{E}(X) = \sum x_i p_i$

Variance: $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	$P(X = x)$	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Uniform distribution over 1, 2, …, n $\mathrm{U}(n)$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2-1)$
Geometric distribution $\mathrm{Geo}(p)$	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $\mathrm{Po}(\lambda)$	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \dfrac{(O_i - E_i)^2}{E_i} \sim \chi_v^2$

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum (x_i - \overline{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}, \; S_{yy} = \sum (y_i - \overline{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},$$

$$S_{xy} = \sum (x_i - \overline{x})(y_i - \overline{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Product moment correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \dfrac{\sum x_i y_i - \dfrac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \dfrac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

The regression coefficient of y on x is $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

Spearman's rank correlation coefficient: $r_s = 1 - \dfrac{6\sum d_i^2}{n(n^2 - 1)}$

Critical values for the product moment correlation coefficient, r

1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999
4	0.9000	0.9500	0.9800	0.9900
5	0.8054	0.8783	0.9343	0.9587
6	0.7293	0.8114	0.8822	0.9172
7	0.6694	0.7545	0.8329	0.8745
8	0.6215	0.7067	0.7887	0.8343
9	0.5822	0.6664	0.7498	0.7977
10	0.5494	0.6319	0.7155	0.7646
11	0.5214	0.6021	0.6851	0.7348
12	0.4973	0.5760	0.6581	0.7079
13	0.4762	0.5529	0.6339	0.6835
14	0.4575	0.5324	0.6120	0.6614
15	0.4409	0.5140	0.5923	0.6411
16	0.4259	0.4973	0.5742	0.6226
17	0.4124	0.4821	0.5577	0.6055
18	0.4000	0.4683	0.5425	0.5897
19	0.3887	0.4555	0.5285	0.5751
20	0.3783	0.4438	0.5155	0.5614
21	0.3687	0.4329	0.5034	0.5487
22	0.3598	0.4227	0.4921	0.5368
23	0.3515	0.4132	0.4815	0.5256
24	0.3438	0.4044	0.4716	0.5151
25	0.3365	0.3961	0.4622	0.5052
26	0.3297	0.3882	0.4534	0.4958
27	0.3233	0.3809	0.4451	0.4869
28	0.3172	0.3739	0.4372	0.4785
29	0.3115	0.3673	0.4297	0.4705
30	0.3061	0.3610	0.4226	0.4629
31	0.3009	0.3550	0.4158	0.4556
32	0.2960	0.3494	0.4093	0.4487
33	0.2913	0.3440	0.4032	0.4421
34	0.2869	0.3388	0.3972	0.4357
35	0.2826	0.3338	0.3916	0.4296
36	0.2785	0.3291	0.3862	0.4238
37	0.2746	0.3246	0.3810	0.4182
38	0.2709	0.3202	0.3760	0.4128
39	0.2673	0.3160	0.3712	0.4076
40	0.2638	0.3120	0.3665	0.4026
41	0.2605	0.3081	0.3621	0.3978
42	0.2573	0.3044	0.3578	0.3932
43	0.2542	0.3008	0.3536	0.3887
44	0.2512	0.2973	0.3496	0.3843
45	0.2483	0.2940	0.3457	0.3801
46	0.2455	0.2907	0.3420	0.3761
47	0.2429	0.2876	0.3384	0.3721
48	0.2403	0.2845	0.3348	0.3683
49	0.2377	0.2816	0.3314	0.3646
50	0.2353	0.2787	0.3281	0.3610
51	0.2329	0.2759	0.3249	0.3575
52	0.2306	0.2732	0.3218	0.3542
53	0.2284	0.2706	0.3188	0.3509
54	0.2262	0.2681	0.3158	0.3477
55	0.2241	0.2656	0.3129	0.3445
56	0.2221	0.2632	0.3102	0.3415
57	0.2201	0.2609	0.3074	0.3385
58	0.2181	0.2586	0.3048	0.3357
59	0.2162	0.2564	0.3022	0.3328
60	0.2144	0.2542	0.2997	0.3301

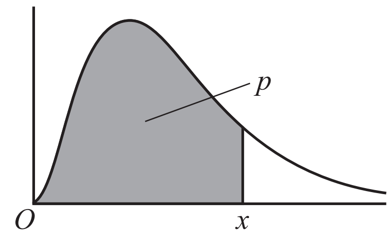
Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%
n				
1	-	-	-	-
2	-	-	-	-
3	-	-	-	-
4	1.0000	-	-	-
5	0.9000	1.0000	1.0000	-
6	0.8286	0.8857	0.9429	1.0000
7	0.7143	0.7857	0.8929	0.9286
8	0.6429	0.7381	0.8333	0.8810
9	0.6000	0.7000	0.7833	0.8333
10	0.5636	0.6485	0.7455	0.7939
11	0.5364	0.6182	0.7091	0.7545
12	0.5035	0.5874	0.6783	0.7273
13	0.4835	0.5604	0.6484	0.7033
14	0.4637	0.5385	0.6264	0.6791
15	0.4464	0.5214	0.6036	0.6536
16	0.4294	0.5029	0.5824	0.6353
17	0.4142	0.4877	0.5662	0.6176
18	0.4014	0.4716	0.5501	0.5996
19	0.3912	0.4596	0.5351	0.5842
20	0.3805	0.4466	0.5218	0.5699
21	0.3701	0.4364	0.5091	0.5558
22	0.3608	0.4252	0.4975	0.5438
23	0.3528	0.4160	0.4862	0.5316
24	0.3443	0.4070	0.4757	0.5209
25	0.3369	0.3977	0.4662	0.5108
26	0.3306	0.3901	0.4571	0.5009
27	0.3242	0.3828	0.4487	0.4915
28	0.3180	0.3755	0.4401	0.4828
29	0.3118	0.3685	0.4325	0.4749
30	0.3063	0.3624	0.4251	0.4670
31	0.3012	0.3560	0.4185	0.4593
32	0.2962	0.3504	0.4117	0.4523
33	0.2914	0.3449	0.4054	0.4455
34	0.2871	0.3396	0.3995	0.4390
35	0.2829	0.3347	0.3936	0.4328
36	0.2788	0.3300	0.3882	0.4268
37	0.2748	0.3253	0.3829	0.4211
38	0.2710	0.3209	0.3778	0.4155
39	0.2674	0.3168	0.3729	0.4103
40	0.2640	0.3128	0.3681	0.4051
41	0.2606	0.3087	0.3636	0.4002
42	0.2574	0.3051	0.3594	0.3955
43	0.2543	0.3014	0.3550	0.3908
44	0.2513	0.2978	0.3511	0.3865
45	0.2484	0.2945	0.3470	0.3822
46	0.2456	0.2913	0.3433	0.3781
47	0.2429	0.2880	0.3396	0.3741
48	0.2403	0.2850	0.3361	0.3702
49	0.2378	0.2820	0.3326	0.3664
50	0.2353	0.2791	0.3293	0.3628
51	0.2329	0.2764	0.3260	0.3592
52	0.2307	0.2736	0.3228	0.3558
53	0.2284	0.2710	0.3198	0.3524
54	0.2262	0.2685	0.3168	0.3492
55	0.2242	0.2659	0.3139	0.3460
56	0.2221	0.2636	0.3111	0.3429
57	0.2201	0.2612	0.3083	0.3400
58	0.2181	0.2589	0.3057	0.3370
59	0.2162	0.2567	0.3030	0.3342
60	0.2144	0.2545	0.3005	0.3314

Critical values for the χ^2 distribution

If X has a χ^2 distribution with v degrees of freedom then, for each pair of values of p and v, the table gives the value of x such that

$$P(X \le x) = p.$$



p	0.01	0.025	0.05	0.90	0.95	0.975	0.99	0.995	0.999
$v = 1$	$0.0^{3}1571$	$0.0^{3}9821$	$0.0^{2}3932$	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Mechanics

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Newton's experimental law

Between two smooth spheres $v_1 - v_2 = -e(u_1 - u_2)$

Between a smooth sphere with a fixed plane surface $v = -eu$

Motion in a circle

Tangential velocity is $v = r\dot{\theta}$

Radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

Tangential acceleration is $\dot{v} = r\ddot{\theta}$

Discrete Mathematics

Sorting algorithms

Bubble sort:

Start at the left hand end of the list unless specified otherwise.

Compare the first and second values and swap if necessary. Then compare the (new) second value with the third value and swap if necessary. Continue in this way until all values have been considered.

Fix the last value then repeat with the reduced list until either there is a pass in which no swaps occur or the list is reduced to length 1, then stop.

Shuttle sort:

Start at the left hand end of the list unless specified otherwise.

Compare the second value with the first and swap if necessary, this completes the first pass. Next compare the third value with the second and swap if necessary, if a swap happened shuttle back to compare the (new) second with the first as in the first pass, this completes the second pass.

Next compare the fourth value with the third and swap if necessary, if a swap happened shuttle back to compare the (new) third value with the second as in the second pass (so if a swap happens shuttle back again). Continue in this way for $n - 1$ passes, where n is the length of the list.

Network algorithms

Dijkstra's algorithm

START with a graph *G*. At each vertex draw a box, the lower area for temporary labels, the upper left hand area for the order of becoming permanent and the upper right hand area for the permanent label.

STEP 1 Make the given start vertex permanent by giving it permanent label 0 and order label 1.

STEP 2 For each vertex that is not permanent and is connected by an arc to the vertex that has just been made permanent (with permanent label = P), add the arc weight to P. If this is smaller than the best temporary label at the vertex, write this value as the new best temporary label.

STEP 3 Choose the vertex that is not yet permanent which has the smallest best temporary label. If there is more than one such vertex, choose any one of them. Make this vertex permanent and assign it the next order label.

STEP 4 If every vertex is now permanent, or if the target vertex is permanent, use 'trace back' to find the routes or route, then STOP; otherwise return to STEP 2.

Prim's algorithm (graphical version)

START with an arbitrary vertex of *G*.

STEP 1 Add an edge of minimum weight joining a vertex already included to a vertex not already included.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Prim's algorithm (tabular version)

START with a table (or matrix) of weights for a connected weighted graph.

STEP 1 Cross through the entries in an arbitrary row, and mark the corresponding column.

STEP 2 Choose a minimum entry from the uncircled entries in the marked column(s).

STEP 3 If no such entry exists STOP; otherwise go to STEP 4.

STEP 4 Circle the weight w_{ij} found in STEP 2; mark column *i*; cross through row *i*.

STEP 5 Return to STEP 2.

Kruskal's algorithm

START with all the vertices of *G*, but no edges; list the edges in increasing order of weight.

STEP 1 Add an edge of *G* of minimum weight in such a way that no cycles are created.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Additional Pure Mathematics

Vector product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$, where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.