

AS Level Further Mathematics A Y533 Mechanics Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

OCR supplied materials:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A
- Scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

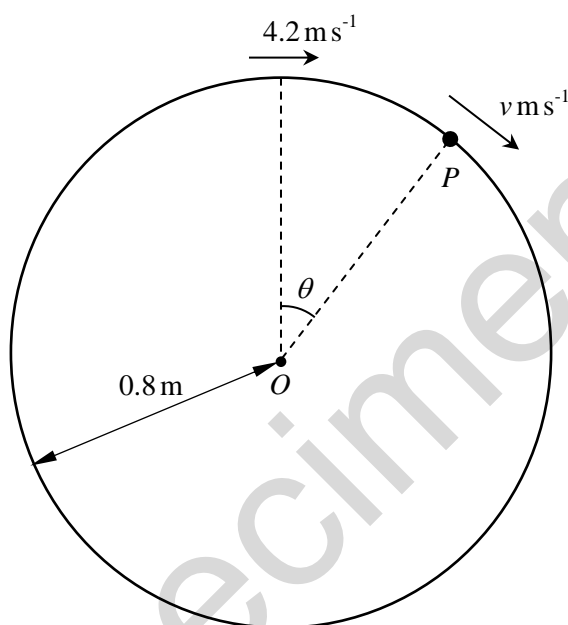
Answer **all** the questions.

- 1** A roundabout in a playground can be modeled as a horizontal circular platform with centre O . The roundabout is free to rotate about a vertical axis through O . A child sits without slipping on the roundabout at a horizontal distance of 1.5 m from O and completes one revolution in 2.4 seconds.

(i) Calculate the speed of the child. [3]

(ii) Find the magnitude and direction of the acceleration of the child. [3]

2



A smooth wire is shaped into a circle of centre O and radius 0.8 m . The wire is fixed in a vertical plane. A small bead P of mass 0.03 kg is threaded on the wire and is projected along the wire from the highest point with a speed of 4.2 m s^{-1} . When OP makes an angle θ with the upward vertical the speed of P is $v\text{ m s}^{-1}$ (see diagram).

(i) Show that $v^2 = 33.32 - 15.68\cos\theta$. [4]

(ii) Prove that the bead is never at rest. [1]

(iii) Find the maximum value of v . [2]

- 3 (i) Write down the dimension of density. [1]

The workings of an oil pump consist of a right, solid cylinder which is partially submerged in oil. The cylinder is free to oscillate along its central axis which is vertical. If the base area of the pump is 0.4 m^2 and the density of the oil is 920 kg m^{-3} then the period of oscillation of the pump is 0.7 s .

A student assumes that the period of oscillation of the pump is dependent only on the density of the oil, ρ , the acceleration due to gravity, g , and the surface area, A , of the circular base of the pump. The student attempts to test this assumption by stating that the period of oscillation, T , is given by $T = C\rho^\alpha g^\beta A^\gamma$ where C is a dimensionless constant.

- (ii) Use dimensional analysis to find the values of α , β and γ . [4]

- (iii) Hence give the value of C to 3 significant figures. [2]

- (iv) Comment, with justification, on the assumption made by the student that the formula for the period of oscillation of the pump was dependent on only ρ , g and A . [2]

- 4 A car of mass 1250 kg experiences a resistance to its motion of magnitude $kv^2 \text{ N}$, where k is a constant and $v \text{ m s}^{-1}$ is the car's speed. The car travels in a straight line along a horizontal road with its engine working at a constant rate of $P \text{ W}$. At a point A on the road the car's speed is 15 m s^{-1} and it has an acceleration of magnitude 0.54 m s^{-2} . At a point B on the road the car's speed is 20 m s^{-1} and it has an acceleration of magnitude 0.3 m s^{-2} .

- (i) Find the values of k and P . [7]

The power is increased to 15 kW .

- (ii) Calculate the maximum steady speed of the car on a straight horizontal road. [3]

5



The masses of two spheres A and B are $3m \text{ kg}$ and $m \text{ kg}$ respectively. The spheres are moving towards each other with constant speeds $2u \text{ m s}^{-1}$ and $u \text{ m s}^{-1}$ respectively along the same straight line towards each other on a smooth horizontal surface (see diagram). The two spheres collide and the coefficient of restitution between the spheres is e . After colliding, A and B both move in the same direction with speeds $v \text{ m s}^{-1}$ and $w \text{ m s}^{-1}$, respectively.

(i) Find an expression for v in terms of e and u . [6]

(ii) Write down unsimplified expressions in terms of e and u for

(a) the total kinetic energy of the spheres before the collision, [1]

(b) the total kinetic energy of the spheres after the collision. [2]

(iii) Given that the total kinetic energy of the spheres after the collision is λ times the total kinetic energy before the collision, show that

$$\lambda = \frac{27e^2 + 25}{52}.$$

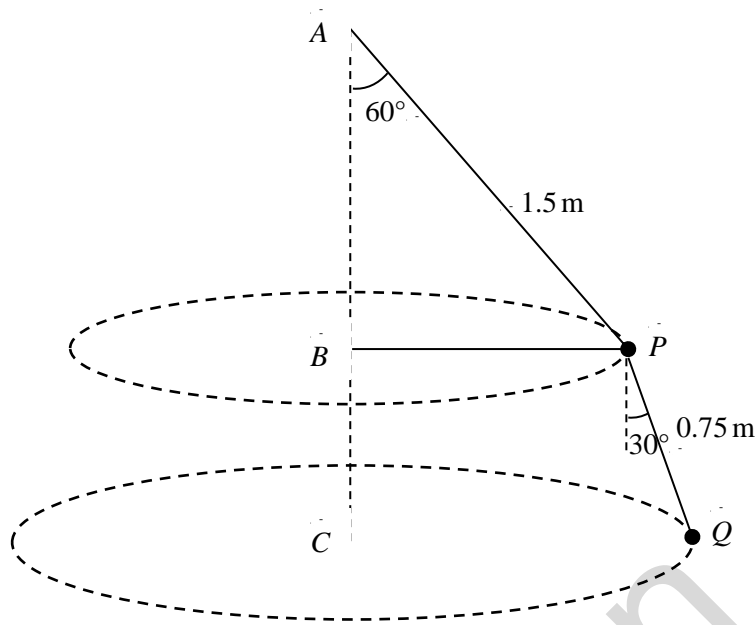
[3]

(iv) Comment on the cases when

(a) $\lambda = 1$,

(b) $\lambda = \frac{25}{52}$.

[3]



The fixed points A , B and C are in a vertical line with A above B and B above C . A particle P of mass 2.5 kg is joined to A , to B and to a particle Q of mass 2 kg, by three light rods where the length of rod AP is 1.5 m and the length of rod PQ is 0.75 m. Particle P moves in a horizontal circle with centre B . Particle Q moves in a horizontal circle with centre C at the same constant angular speed ω as P , in such a way that A , B , P and Q are coplanar. The rod AP makes an angle of 60° with the downward vertical, rod PQ makes an angle of 30° with the downward vertical and rod BP is horizontal (see diagram).

- (i) Find the tension in the rod PQ . [2]
- (ii) Find ω . [3]
- (iii) Find the speed of P . [1]
- (iv) Find the tension in the rod AP . [3]
- (vi) Hence find the magnitude of the force in rod BP .
Decide whether this rod is under tension or compression. [4]

END OF QUESTION PAPER

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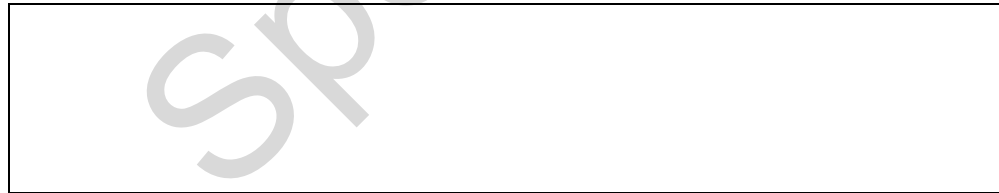
AS Level Further Mathematics A

Y533 Mechanics

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60



This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for AS Level Further Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AOs	Guidance
1	(i)	$\frac{2\pi}{\omega} = 2.4$ Using $v = r\omega = 3.93 \text{ ms}^{-1}$	B1 M1A1 [3]	1.1a 1.1, 1.1	3.9269908...
1	(ii)	$a = \frac{v^2}{r} = \frac{\left(\frac{5}{4}\pi\right)^2}{1.5}$ $a = 10.3$ The direction is towards the centre O	M1 A1 B1 [3]	1.1 1.1 1.1	Applying either $\frac{v^2}{r}$ or $r\omega^2$ with their v . Magnitude, accept 10.3 ms^{-2} May be awarded from incorrect a .
2	(i)	Initial kinetic energy $= \frac{1}{2}(0.03)(4.2)^2$ At angle θ , potential energy = $-0.03g(0.8 - 0.8\cos\theta)$ $\frac{1}{2}(0.03)(4.2)^2 = -0.03g(0.8 - 0.8\cos\theta) + \frac{1}{2}(0.03)v^2$ $v^2 = 33.32 - 15.68\cos\theta$	B1 B1 M1 E1 [4]	1.1 3.3 3.4 2.1	Allow m used instead of 0.03 Allow m used instead of 0.03 Attempt at conservation of mechanical energy (correct number of terms) and rearranging to make v^2 the subject www; AG at least one intermediate step must be shown
2	(ii)	e.g. $v = 0 \Rightarrow \cos\theta = 2.125 > 1$ so never at rest	E1 [1]	2.2a	e.g. The speed of the bead at any point below the top of the circle must be higher than its initial speed because the bead has lost PE and thus gained KE
2	(iii)	$v^2 = 33.32 - 15.68 \times (-1)$ $v = 7$	M1 A1 [2]	1.1 1.1	Setting $\cos\theta = -1$ and solving for v

Question		Answer	Marks	AOs	Guidance
3	(i)	$\rho = ML^{-3}$	B1 [1]	1.2	Allow ML^{-3}
3	(ii)	$T = (ML^{-3})^{\alpha} (LT^{-2})^{\beta} (L^2)^{\gamma} = M^{\alpha} L^{-3\alpha+\beta+2\gamma} T^{-2\beta}$ $M: \alpha = 0$ $L: -3\alpha + \beta + 2\gamma = 0$ $T: -2\beta = 1$ $\beta = -\frac{1}{2}$ $\gamma = \frac{1}{4}$	M1 B1 M1 A1 [4]	3.3 1.1 3.4 1.1	For $\alpha = 0$ Setting up equations and solving simultaneously to find β and γ For β and γ correct
	(iii)	$T = C g^{-\frac{1}{2}} A^{\frac{1}{4}}$ 2.76	M1 A1 [2]	1.1 3.5c	Substituting values into their equation for T $C = 2.755472\dots$
3	(iv)	e.g. According to this model the period is independent of the density e.g. Because $\alpha = 0$	*E1 dep*E1 [2]	2.2b 3.5b	e.g. It is possible that the period is dependent on another (unknown) quantity e.g. because the value of C may depend on a variable that was not considered

Question		Answer	Marks	AOs	Guidance	
5	(i)	$3m(2u) + m(-u) = 3mv + mw$ $v - w = -e(2u - (-u))$ $v = \frac{1}{4}u(5 - 3e)$	<p>*M1</p> <p>A1</p> <p>*M1</p> <p>A1</p> <p>dep*M1</p> <p>A1</p> <p>[6]</p>	<p>1.1a</p> <p>1.1</p> <p>1.1a</p> <p>1.1</p> <p>3.1b</p> <p>1.1</p>	<p>Attempt at use of conservation of linear momentum</p> <p>Attempt at use of restitution equation, must be correct way round</p> <p>Must be consistent with directions used for conservation of linear momentum</p> <p>Solving simultaneous equations</p>	
5	(ii)	(a)	$T_{before} = \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}m(-u)^2$ <p>[1]</p>	B1 1.1		
5	(ii)	(b)	$T_{after} = \frac{1}{2}(3m)\left(\frac{u}{4}\right)^2(5 - 3e)^2 + \frac{1}{2}(m)\left(\frac{u}{4}\right)^2(5 + 9e)^2$ <p>[2]</p>	B1FT B1FT 1.1 1.1	B1 for each term correct for their part (i)	
5	(iii)	$\frac{13}{2}\lambda mu^2 = \frac{3}{32}mu^2(5 - 3e)^2 + \frac{1}{32}mu^2(5 + 9e)^2$ $\lambda = \frac{27e^2 + 25}{52}$	B1FT M1 E1 [3]	3.4 1.1 1.1	$\lambda T_{before} = T_{after}$ Rearrange to make λ the subject AG cancelling of mu^2 factor must be seen and expansion of brackets	
5	(iv)	(a)	$\lambda = 1 (\Rightarrow e = 1)$ so the collision is perfectly elastic	B1	2.4	oe e.g. there is no loss in kinetic energy language must be precise
		(b)	$\lambda = \frac{25}{52} \Rightarrow e = 0$ so the collision is perfectly inelastic	M1 A1 [3]	1.1 2.4	Substitute λ and solve for e oe e.g. the particles coalesce language must be precise

Question		Answer	Marks	AOs	Guidance
6	(i)	$T_{PQ} \cos 30 = 2g$ $T_{PQ} = 22.6\text{N}$	M1 A1 [2]	3.3 1.1	Resolving vertically for Q 22.632130...
6	(ii)	$r = 1.5 \sin 60 + 0.75 \sin 30$ $T_{PQ} \sin 30 = 2r\omega^2$ $\omega = 1.84 \text{ rad s}^{-1}$	B1 M1 A1 [3]	3.1b 3.3 1.1	May be implied in N2L for Q N2L for Q with their r 1.8384425...
6	(iii)	$v_P = 2.39 \text{ m s}^{-1}$	B1FT [1]	1.1	(1.5 sin 60)(their ω)
6	(iv)	$T_{AP} \cos 60 = 2.5g + T_{PQ} \cos 30$ $T_{AP} = 88.2\text{N}$	M1 M1 A1 [3]	3.1b 1.1 1.1	Resolving vertically for P Substituting cv for T_{PQ} from (i) and solving for T_{AP}
6	(v)	$T_{BP} + T_{AP} \sin 60 - T_{PQ} \sin 30 = 2.5(1.5 \sin 60)\omega^2$ $T_{BP} = -54.1\text{N}$ so $ T_{BP} = 54.1$ $T_{BP} < 0 \therefore$ rod BP is under compression	M1 A1 A1 A1FT [4]	3.1b 1.1 1.1 3.2a	N2L for P (4 terms) FT the sign of T_{BP} -54.0909226...

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1(i)	3				3
1(ii)	3				3
2(i)	1	1		2	4
2(ii)		1			1
2(iii)	2				2
3(i)	1				1
3(ii)	3			3	6
3(iii)		1		1	2
4(i)	5			2	7
4(ii)	1	1		1	3
5(i)	5		1		6
5(ii)(a)	1				1
5(ii)(b)	2				2
5(iii)	2			1	3
5(iv)	1	2			3
6(i)	1			1	2
6(ii)	1		1	1	3
6(iii)	1				1
6(iv)	2		1		3
6(v)	2		2		4
6(vi)					
Totals	37	6	5	12	60

PS = Problem Solving

M = Modelling

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AS Level Further Mathematics A

Y533 Mechanics

Printed Answer Booklet

Date – Morning/Afternoon

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First name

Last name

Centre number

Candidate number

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3(i)	
3(ii)	

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3(iii)	
3(iv)	

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5(i)	
5(ii)(a)	
5(ii)(b)	

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5(iii)	
5(iv)(a)	
(b)	

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6(i)

6(ii)

6(iii)

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