

## A Level Further Mathematics A Y542 Statistics Sample Question Paper

### Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

#### OCR supplied materials:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

#### You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A
- Scientific or graphical calculator



#### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

#### INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1** The table below shows the typical stopping distances  $d$  metres for a particular car travelling at  $v$  miles per hour.

$v$	20	30	40	50	60	70
$d$	13	24	36	52	72	94

- (i) State each of the following words that describe the variable  $v$ .

Independent    Dependent    Controlled    Response

[1]

- (ii) Calculate the equation of the regression line of  $d$  on  $v$ .

[2]

- (iii) Use the equation found in part (ii) to estimate the typical stopping distance when this car is travelling at 45 miles per hour.

[1]

It is given that the product moment correlation coefficient for the data is 0.990 correct to three significant figures.

- (iv) Explain whether your estimate found in part (iii) is reliable.

[2]

- 2** The mass  $J$  kg of a bag of randomly chosen Jersey potatoes is a normally distributed random variable with mean 1.00 and standard deviation 0.06. The mass  $K$  kg of a bag of randomly chosen King Edward potatoes is an independent normally distributed random variable with mean 0.80 and standard deviation 0.04.

- (i) Find the probability that the total mass of 6 bags of Jersey potatoes and 8 bags of King Edward potatoes is greater than 12.70 kg.

[3]

- (ii) Find the probability that the mass of one bag of King Edward potatoes is more than 75% of the mass of one bag of Jersey potatoes.

[3]

- 3** A game is played as follows. A fair six-sided dice is thrown once. If the score obtained is even, the amount of money, in £, that the contestant wins is half the score on the dice, otherwise it is twice the score on the dice.

- (i) Find the probability distribution of the amount of money won by the contestant.

[3]

- (ii) The contestant pays £5 for every time the dice is thrown.

Find the standard deviation of the loss made by the contestant in 120 throws of the dice.

[5]

- 4 A psychologist investigated the scores of pairs of twins on an aptitude test. Seven pairs of twins were chosen randomly, and the scores are given in the following table.

Elder twin	65	37	60	79	39	40	88
Younger twin	58	39	61	62	50	26	84

- (i) Carry out an appropriate Wilcoxon test at the 10% significance level to investigate whether there is evidence of a difference in test scores between the elder and the younger of a pair of twins. [6]
- (ii) Explain the advantage in this case of a Wilcoxon test over a sign test. [1]

- 5 The number of goals scored by the home team in a randomly chosen hockey match is denoted by  $X$ .

- (i) In order for  $X$  to be modelled by a Poisson distribution it is assumed that goals scored are random events. State two other conditions needed for  $X$  to be modelled by a Poisson distribution in this context. [2]

Assume now that  $X$  can be modelled by the distribution  $Po(1.9)$ .

- (ii) (a) Write down an expression for  $P(X = r)$ . [1]  
 (b) Hence find  $P(X = 3)$ . [1]
- (iii) Assume also that the number of goals scored by the away team in a randomly chosen hockey match has an independent Poisson distribution with mean  $\lambda$  between 1.31 and 1.32. Find an estimate for the probability that more than 3 goals are scored altogether in a randomly chosen match. [4]

- 6 A bag contains 3 green counters, 3 blue counters and  $w$  white counters. Counters are selected at random, one at a time, with replacement, until a white counter is drawn. The total number of counters selected, including the white counter, is denoted by  $X$ .

- (i) In the case when  $w = 2$ ,
- (a) write down the distribution of  $X$ , [1]  
 (b) find  $P(3 < X \leq 7)$ . [2]
- (ii) In the case when  $E(X) = 2$ , determine the value of  $w$ . [2]
- (iii) In the case when  $w = 2$  and  $X = 6$ , find the probability that the first five counters drawn alternate in colour. [2]

- 7 Sweet pea plants grown using a standard plant food have a mean height of 1.6 m. A new plant food is used for a random sample of 49 randomly chosen plants and the heights,  $x$  metres, of this sample can be summarised by the following.

$$\begin{aligned}n &= 49 \\ \Sigma x &= 74.48 \\ \Sigma x^2 &= 120.8896\end{aligned}$$

Test, at the 5% significance level, whether, when the new plant food is used, the mean height of sweet pea plants is less than 1.6 m. [9]

- 8 A continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} 0.8e^{-0.8x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

- (i) Find the mean and variance of  $X$ . [4]

The lifetime of a certain organism is thought to have the same distribution as  $X$ . The lifetimes in days of a random sample of 60 specimens of the organism were found. The observed frequencies, together with the expected frequencies correct to 3 decimal places, are given in the table.

Range	$0 \leq x < 1$	$1 \leq x < 2$	$2 \leq x < 3$	$3 \leq x < 4$	$x \geq 4$
Observed	24	22	10	3	1
Expected	33.040	14.846	6.671	2.997	2.446

- (ii) Show how the expected frequency for  $1 \leq x < 2$  is obtained. [4]

- (iii) Carry out a goodness of fit test at the 5% significance level. [7]

- 9 The continuous random variable  $X$  has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{16}x^2 & 0 \leq x \leq 4, \\ 1 & x > 4. \end{cases}$$

- (i) The random variable  $Y$  is defined by  $Y = \frac{1}{X^2}$ . Find the cumulative distribution function of  $Y$ . [5]

- (ii) Show that  $E(Y)$  is not defined. [4]

### END OF QUESTION PAPER

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**...day June 20XX – Morning/Afternoon**

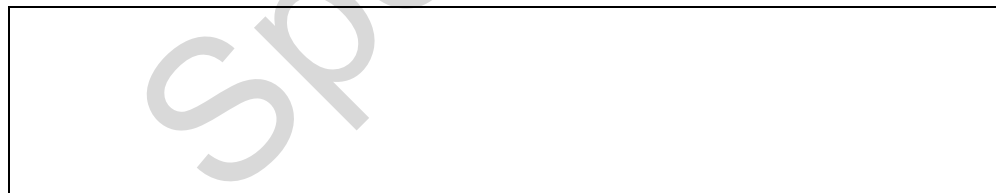
**A Level Further Mathematics A**

**Y542 Statistics**

**SAMPLE MARK SCHEME**

**Duration:** 1 hour 30 minutes

**MAXIMUM MARK    75**



**This document consists of 16 pages**

## Text Instructions

## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

## 2. Subject-specific Marking Instructions for A Level Further Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

### **M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### **A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### **B**

Mark for a correct result or statement independent of Method marks.

### **E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.



Question		Answer	Marks	AO	Guidance
1	(i)	Independent and controlled	<b>B1</b> [1]	<b>1.2</b>	Both, no others
1	(ii)	$d = 1.61v - 24.1$	<b>B2</b> [2]	<b>1.1</b> <b>3.3</b>	All correct including letters, 3 s.f. BC <b>B1</b> Numbers right but not letters [ $d = 1.614v - 24.143$ ]
1	(iii)	$d = 1.61 \times 45 - 24.1 = 48$ to the nearest whole number	<b>B1</b> [1]	<b>3.4</b>	awrt 48.5
1	(iv)	Yes as $r$ is close to 1 and 45 is within data range	<b>E1</b> <b>E1</b> [2]	<b>3.5a</b> <b>3.5b</b>	Yes with one reason Second reason
2	(i)	$\sum J + \sum K \sim N(12.4, 0.0344)$ $P(> 12.7) = 1 - 0.9471 = 0.0529$	<b>M1</b> <b>A1</b> <b>A1</b> [3]	<b>1.1a</b> <b>1.1</b> <b>1.1</b>	Consider the sum $\sim N(12.4, \dots)$ Standard deviation or variance correct awrt 0.053 BC 0.232 or 0.68: <b>M1A0</b>
2	(ii)	$K - 0.75J \sim N(0.05, 0.003625)$ $P(> 0) = \Phi(0.08305) = 0.7969$	<b>M1</b> <b>A1</b> <b>A1</b> [3]	<b>1.1a</b> <b>1.1</b> <b>1.1</b>	Or $4K - 3J \sim N(0.2, \dots)$ Standard deviation or variance correct 0.0043 or 0.085: <b>M1A0</b> awrt 0.797 BC

Question		Answer	Marks	AO	Guidance												
3	(i)	<table border="1"> <tr> <td><math>x</math> (£)</td> <td>1</td> <td>2</td> <td>3</td> <td>6</td> <td>10</td> </tr> <tr> <td><math>P(X = x)</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{2}{6}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> </tr> </table>	$x$ (£)	1	2	3	6	10	$P(X = x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	<b>M1</b>	<b>3.1b</b>	$x$ -values correct
		$x$ (£)	1	2	3	6	10										
$P(X = x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$												
			<b>A1</b> <b>A1</b> <b>[3]</b>	<b>1.1</b> <b>1.1</b>	At least 2 probabilities correct All correct												
3	(ii)	$\sum xP(x) = \frac{1}{6} + \frac{4}{6} + \frac{3}{6} + \frac{6}{6} + \frac{10}{6} = 4$	<b>B1</b>	<b>2.2a</b>	For dismissing the £5 loss, or using profit $y$ :												
					<table border="1"> <tr> <td><math>y</math> (£)</td> <td>-4</td> <td>-3</td> <td>-2</td> <td>1</td> <td>5</td> </tr> <tr> <td><math>P(Y = y)</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{2}{6}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> </tr> </table>	$y$ (£)	-4	-3	-2	1	5	$P(Y = y)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
		$y$ (£)	-4	-3	-2	1	5										
		$P(Y = y)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$										
						giving $\sum yP(y) = -1$ and											
				$\sum y^2P(y) = \frac{32}{3}$													
			<b>M1</b>	<b>1.1</b>	Allow their value of $\mu$												
		$\sum x^2P(x) - \mu^2 = \frac{1}{6} + \frac{8}{6} + \frac{9}{6} + \frac{36}{6} + \frac{100}{6} - \mu^2$ $= \frac{77}{3} - \mu^2$ $= 9\frac{2}{3}$	<b>A1</b>	<b>1.1</b>													
		Therefore for 120 games the standard deviation is	<b>M1</b>	<b>2.2a</b>	Multiply by 120 and take $\sqrt{\quad}$												
		$\sqrt{120 \times 9\frac{2}{3}} = 34.1$	<b>A1FT</b>	<b>1.1</b>	In range [34(.0), 35.1]												
			<b>[5]</b>														

Question		Answer	Marks	AO	Guidance																						
4	(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>7</td><td>2</td><td>1</td><td>17</td><td>11</td><td>14</td><td>4</td></tr> <tr><td>4</td><td>2</td><td>1</td><td>7</td><td>6</td><td>5</td><td>3</td></tr> <tr><td>+</td><td>-</td><td>-</td><td>+</td><td>-</td><td>+</td><td>+</td></tr> </table> <p> <math>H_0</math> : population median difference = 0  <math>H_1</math> : population median difference <math>\neq</math> 0  <math>P = 4 + 7 + 5 + 3 = 19</math>  <math>Q = 1 + 2 + 6 = 9</math>  <math>T = 8</math>  <math>T_{crit} = 3</math>; <math>8 &gt; 3</math>            Do not reject <math>H_0</math>. Insufficient evidence of a difference in test scores         </p>	7	2	1	17	11	14	4	4	2	1	7	6	5	3	+	-	-	+	-	+	+	<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>A1FT</b></p> <p>[6]</p>	<p><b>1.1</b></p> <p><b>2.5</b></p> <p><b>3.3</b></p> <p><b>3.4</b></p> <p><b>1.1</b></p> <p><b>2.2b</b></p> <p><b>3.5b</b></p>	<p>Calculate differences, rank them and attach signs</p> <p>Hypotheses correctly stated</p> <p><math>P</math> or <math>Q</math> correct</p> <p>Both <math>P</math> and <math>Q</math> seen, <math>T</math> correct</p> <p>Comparison with correct <math>T_{crit}</math></p> <p>Correct conclusion, in context, acknowledge uncertainty</p> <p>FT their <math>T</math> but not their <math>T_{crit}</math></p>	<p>Follow through with correct signs and ranks from incorrect differences</p> <p><b>SC3</b>: Two-sample, max 3/6</p>
7	2	1	17	11	14	4																					
4	2	1	7	6	5	3																					
+	-	-	+	-	+	+																					
4	(ii)	Uses magnitude of differences oe	<p><b>B1</b></p> <p>[1]</p>																								
5	(i)	<p>Goals are scored independently</p> <p>Goals are scored at uniform rate</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p>[2]</p>	<p><b>1.2</b></p> <p><b>1.2</b></p>	<p>Not “singly”</p> <p>Must be in context</p>	<p>Allow “constant average rate” but not “constant rate”.</p> <p><b>B0</b> for any answer that implies fixed numbers in given time.</p> <p><b>B0</b> for “events must occur randomly”, “independently”, “singly” or “at constant rate” oe</p>																					

Question			Answer	Marks	AO	Guidance
5	(ii)	(a)	$P(X = r) = e^{-1.9} \frac{1.9^r}{r!}$	B1 [1]	1.1	Must be seen
5	(ii)	(b)	$P(X = 3) = 0.171$	B1 [1]	1.1	
5	(iii)		Total $\sim P_o(1.9 + \lambda)$ $\lambda = (1.9 + 1.31)$ , $P(> 3) = 0.399\dots$ $\lambda = (1.9 + 1.32)$ , $P(> 3) = 0.401\dots$ $0.399\dots < 0.4$ and $0.401\dots > 0.4$ , hence a reasonable estimate is 0.4	M1 M1 A1 E1 [4]	2.2a 3.1b 1.1 3.2a	Use $1.9 + \lambda$ Evaluate RH tail probability for 1.31 and 1.32 Both evaluations correct BC BC
6	(i)	(a)	$X \sim \text{Geo} \left(\frac{1}{4}\right)$	B1 [1]	2.5	Accept $\text{Geo} \left(\frac{1}{4}\right)$ oe
6	(i)	(b)	$\left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^7$  $= \frac{4725}{16384}$	M1 A1 [2]	1.1 1.1	Expression of the form $\left(\frac{3}{4}\right)^a - \left(\frac{3}{4}\right)^b$ with $a < b$ awrt 0.288
6	(ii)		$E(X) = 2 \Rightarrow p = \frac{1}{2}$ Hence $w = 6$	M1 A1 [2]	2.2a 2.2a	
6	(iii)		$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$	M1 A1 [2]	3.1a 1.1	Or, e.g. $\left(\frac{3}{4}\right) \times \left(\frac{3}{8}\right)^4 \div \left(\frac{3}{4}\right)^5$

Question	Answer	Marks	AO	Guidance
7	$\mu = \bar{x} = 1.52$ $\hat{\sigma}^2 = \frac{49}{48} \left( \frac{120.8896}{49} - 1.52^2 \right)$ $= 0.16$ $H_0 : \mu = 1.6$ $H_1 : \mu < 1.6$ $p = 0.0808 \text{ or } z = \frac{1.52 - 1.6}{\sqrt{\frac{0.16}{49}}} = -1.4$ <p>0.0808 &gt; 0.05 or <math>-1.4 &gt; -1.645</math>            Do not reject <math>H_0</math>            Insufficient evidence that height of plants using new plant food is less than 1.6</p>	<p><b>B1</b> <b>B1</b>  <b>B1</b> <b>B1</b>  <b>M1</b> <b>A1</b>   <b>A1</b> <b>M1</b> <b>A1FT</b></p> <p>[9]</p>	<p><b>3.1b</b> <b>3.3</b>  <b>1.1</b> <b>2.5</b>  <b>2.1</b> <b>3.4</b>   <b>1.1</b> <b>1.1</b> <b>2.2b</b></p>	<p>1.52 seen            Biased estimate (0.1567) <b>B0</b> but can get all subsequent marks            Hypotheses both correct, <b>B2</b>. One error, <b>B1</b>, but use of <math>x</math> or <math>\bar{x}</math> or 1.52 is <b>B0B0</b>            Evidence for 49 divisor needed (<i>see notes</i>)  <math>p = 0.0808</math> or <math>z = -1.4</math> seen, allow +1.4            BC            Allow <math>1.4 &lt; 1.645</math> only if consistent            Correct method, comparison and conclusion            Contextualised, acknowledge uncertainty, needs double negative [<i>not</i> “evidence that height is 1.6”]. FT on <math>z</math>. Do not award final <b>M1A1</b> if <i>either</i> 49 divisor missing <i>or</i> hypotheses given in terms of 1.52</p> <p>(<math>\alpha</math>) Unless wrong working is seen, <math>p = 0.0808</math> or <math>z = -1.4</math> automatically gets <b>M1A1</b> and (unless hypotheses are given in terms of 1.52) automatically qualifies for <b>A1M1A1FT</b>            (<math>\beta</math>) If neither <math>p = 0.0808</math> or <math>z = -1.4</math> is seen, all of the last 5 marks depend on seeing <i>either</i> <math>N\left(1.6, \frac{0.16}{49}\right)</math> oe, or <math>\frac{1.52 - 1.6}{\sqrt{\frac{0.16}{49}}}</math>. Either of these seen but with square root errors can get <b>M1A0A1M1A1FT</b>            (<math>\gamma</math>) “cdfnorm” notation, or similar, with wrong <math>p</math> or <math>z</math> does <i>not</i> qualify for <b>M1A0A1</b> but can get last <b>M1A1FT</b> provided 49 is seen to be used and hypotheses not stated in terms of 1.52. “cdfnorm” notation with correct <math>p</math> or <math>z</math> can get full marks.</p>

Question		Answer	Marks	AO	Guidance																
8	(i)	$\mu = \int_0^{\infty} 0.8xe^{-0.8x} dx = 1.25$  $E(X^2) = \int_0^{\infty} 0.8x^2e^{-0.8x} dx [= 3.125]$  $\text{Var}(X) = 3.125 - 1.25^2$ $= 1.5625$	M1 A1	1.1a 1.1	Attempt $\int xf(x) dx$  Obtain 1.25 or exact equivalent	BC															
		$\text{Var}(X) = 3.125 - 1.25^2$ $= 1.5625$	M1 A1	1.1 1.1	Attempt $\int x^2f(x) dx - \mu^2$  Obtain $\frac{25}{16}$ or exact equivalent	or awrt 1.56 BC															
	(ii)	$P(1 \leq x < 2) = \int_1^2 0.8e^{-0.8x} dx$ $= 0.247432$ (6 s.f.) There are 60 specimens, so the expected frequency is $0.247432 \times 60 = 14.846$	M1 E1 A1 E1	1.1 2.1 1.1 2.2a	Correct pdf Integrate between 1 and 2 Correct answer, allow 3 s.f. Multiply probability by 60 and correctly obtain given answer AG	Requires clear use of notation BC															
	(iii)	$H_0$ : data consistent with distribution $H_1$ : data not consistent Combine cells to get <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>O</th> <th>E</th> <th><math>(O-E)^2 / E</math></th> </tr> </thead> <tbody> <tr> <td>24</td> <td>33.040</td> <td>2.4734</td> </tr> <tr> <td>22</td> <td>14.846</td> <td>3.4474</td> </tr> <tr> <td>10</td> <td>6.6707</td> <td>1.6613</td> </tr> <tr> <td>4</td> <td>5.4431</td> <td>0.3826</td> </tr> </tbody> </table> $\sum \frac{(O-E)^2}{E} = 7.965$ $\chi_3^2(0.95) = 7.815$ and $7.965 > 7.815$ Reject $H_0$ . Evidence that the data is not consistent with distribution	O	E	$(O-E)^2 / E$	24	33.040	2.4734	22	14.846	3.4474	10	6.6707	1.6613	4	5.4431	0.3826	B1 M1 M1 A1 A1 B1 A1FT	2.5 1.1a 1.1 1.1 3.4 1.1 2.2b	Or equivalent  Combine last two cells Calculate $\frac{(O-E)^2}{E}$ for at least one cell At least two $\frac{(O-E)^2}{E}$ values correct  $\chi^2$ in range [7.96, 7.97]  Comparison with 7.815 State not consistent with distribution FT on numerical errors only	BC
O	E	$(O-E)^2 / E$																			
24	33.040	2.4734																			
22	14.846	3.4474																			
10	6.6707	1.6613																			
4	5.4431	0.3826																			
			[4] [4] [7]																		

Question		Answer	Marks	AO	Guidance
9	(i)	$P(Y \leq y) = P\left(\frac{1}{X^2} \leq y\right)$ $= P\left(X \geq \frac{1}{\sqrt{y}}\right)$ $= 1 - F\left(\frac{1}{\sqrt{y}}\right)$ $= \begin{cases} 1 - \frac{1}{16y} & y > \frac{1}{16}, \\ 0 & \text{otherwise.} \end{cases}$	M1	1.1a	Attempt to write $F_y$ in terms of $X$
			E1	2.1	Make $X$ the subject
			M1	2.1	$1 - F$ (inverse function)
			E1	3.1a	$1 - \frac{1}{16y}$ correct, www
			B1	1.1	0 and ranges correct (independent)
			[5]		

Question		Answer	Marks	AO	Guidance
9	(ii)	PDF of $y$ is $\frac{1}{16y^2}$ $\int_{\frac{1}{16}}^{\infty} \frac{y}{16y^2} dy$ $= \left[ \frac{1}{16} \ln y \right]_{\frac{1}{16}}^{\infty}$ and $\ln y$ is undefined as $y \rightarrow \infty$	M1	3.1a	Differentiate CDF to find PDF of $Y$
			M1	1.1	Multiply by $y$ and integrate, using their limits
			A1	2.1	Integration must be shown explicitly
			E1	3.2a	Correctly justify given statement
			[4]		A0A0 For “calculator gives math error” or similar
		<b>OR</b> PDF of $x$ is $\frac{1}{8}x$ $E(Y) = E\left(\frac{1}{X^2}\right)$ $= \int_0^4 \frac{1}{x^2} \frac{1}{8} x dx$ $= \left[ \frac{1}{8} \ln x \right]_0^4$ and $\ln x$ is undefined as $x \rightarrow 0$	M1		Differentiate CDF to find PDF of $X$
			M1		Integrate $\frac{1}{x^2} \times \text{PDF}$ , limits 0, 4
			A1		Integration must be shown explicitly
			E1		Correctly justify given statement
			[4]		A0A0 For “calculator gives math error” or similar



## Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
<b>1(i)</b>	1				<b>1</b>
<b>1(ii)</b>	1			1	<b>2</b>
<b>1(iii)</b>				1	<b>1</b>
<b>1(iv)</b>				2	<b>2</b>
<b>2(i)</b>	3				<b>3</b>
<b>2(ii)</b>	3				<b>3</b>
<b>3(i)</b>	2		1		<b>3</b>
<b>3(ii)</b>	3	2			<b>5</b>
<b>4(i)</b>	2	2		2	<b>6</b>
<b>4(ii)</b>				1	<b>1</b>
<b>5(i)</b>	2				<b>2</b>
<b>5(ii)(a)</b>				1	<b>1</b>
<b>5(ii)(b)</b>	1				<b>1</b>
<b>5(iii)</b>	1	1	2		<b>4</b>
<b>6(i)(a)</b>		1			<b>1</b>
<b>6(i)(b)</b>	2				<b>2</b>
<b>6(ii)</b>		2			<b>2</b>
<b>6(iii)</b>	1		1		<b>2</b>
<b>7</b>	3	3	1	2	<b>9</b>
<b>8(i)</b>	4				<b>4</b>
<b>8(ii)</b>	2	2			<b>4</b>
<b>8(iii)</b>	4	2		1	<b>7</b>
<b>9(i)</b>	2	2	1		<b>5</b>
<b>9(ii)</b>	1	1	2		<b>4</b>
<b>Totals</b>	<b>38</b>	<b>18</b>	<b>8</b>	<b>11</b>	<b>75</b>

PS = Problem Solving  
M = Modelling

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## A Level Further Mathematics A

### Y542 Statistics

Printed Answer Booklet

## Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

**OCR supplied materials:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A
- Scientific or graphical calculator



First name

Last name

Centre  
number

Candidate  
number

### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

<b>1(i)</b>	
<b>1(ii)</b>	
<b>1(iii)</b>	
<b>1(iv)</b>	

Specimen

2(i)


2(ii)


Specimen

<b>3(i)</b>	
<b>3(ii)</b>	

Specimen



4(i)	
	4(ii)

<b>5(i)</b>	
<b>5(ii)(a)</b>	
<b>5(ii)(b)</b>	
<b>5(iii)</b>	

Specimen

<b>6(i)</b>	
<b>6(ii)</b>	
<b>6(iii)</b>	

Specimen

7

Specimen

**8(i)****8(ii)**

Specimen

8(iii)

Specimen

9(i)

Specimen

<b>9(ii)</b>	

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