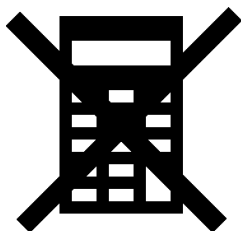AQA, OCR, Edexcel

GCSE

GCSE Maths

Proof

Name:

Guidance

1. Read each question carefully.
2. Don't spend too long on each question.
3. Attempt every question.
4. Always show your workings.

Revise GCSE Maths:
www.MathsMadeEasy.co.uk/gcse-maths-revision/

1. Show that the following statement is true

$$5(3x - 5) - 2(2x + 9) \equiv 11x - 43$$

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(1 mark)

2. Show that the following statement is true

$$(n - 2)^2 - (n - 5)^2 \equiv 3(2n - 7)$$

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(2 marks)

3. Show that the following statement is true

$$(n + 2)^2 - 3(n + 4) \equiv (n + 4)(n - 3) + 4$$

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(2 marks)

4. Show that the following statement is true

$$3(n + 3)(n - 1) - 3(1 - n) \equiv (3n - 3)(n + 4)$$

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(2 marks)

5. Prove that

$$(n + 3)^2 + n(3 - n) - 3(n + 4)$$

is a multiple of 3 for all integer values of n.

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(2 marks)

6. Prove algebraically that the sum of two consecutive numbers is odd.

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(2 marks)

7. Prove algebraically that the sum of the squares of two consecutive multiples of 5 is not a multiple of 10.

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What would the remainder be if this number were divided by 5?

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(2 marks, 1 mark)

8. Tom says that $7x - (2x + 3)(x + 2)$ is always negative.

   Is he correct? Explain your answer.

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   Change a single number in Tom's statement that would lead to a change in your conclusion. Why is this the case?

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   (3 marks, 1 mark)

9. Show that the difference between 14^{20} and 21^2 is a multiple of 7.

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(3 marks)

10. Show that $3^{60} - 25$ is not a prime number.

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(2 marks)

11. Part of a 10x10 1-100 number grid is pictured below.

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

A 2x2 square of numbers is selected.
The following operation is performed:

Difference of the leading diagonal × Difference of the other diagonal

$$(23 - 12) \times (22 - 13) = 11 \times 9 = 99$$

Verify that this is also the case for a different 2x2 square of numbers on the grid.

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By generalising, prove this result for all possible 2x2 squares on the grid.

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(1 mark, 3 marks)

12. The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is a re-arrangement of the general quadratic equation

$$ax^2 + bx + c = 0$$

By completing the square on the general quadratic equation, prove this result.

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(5 marks)