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AQA, OCR, Edexcel

GCSE

GCSE Maths

Proof

Name:

Guidance

1. Read each question carefully.
2. Don't spend too long on each question.
3. Attempt every question.
4. Always show your workings.

Revise GCSE Maths:
www.MathsMadeEasy.co.uk/gcse-maths-revision/

1. Show that the following statement is true

$$5(3x-5)-2(2x+9)\equiv 11x-43$$

$$\begin{aligned}5(3x-5)-2(2x+9) &= 15x-25-4x-18\\ &= 15x-4x-25-18\\ &= 11x-43\end{aligned}$$

(1 mark)

2. Show that the following statement is true

$$(n-2)^2-(n-5)^2\equiv 3(2n-7)$$

$$\begin{aligned}(n-2)^2-(n-5)^2 &= (n-2)(n-2)-(n-5)(n-5)\\ &= n^2-2n-2n+4-(n^2-5n-5n+25)\\ &= n^2-4n+4-(n^2-10n+25)\\ &= n^2-4n+4-n^2+10n-25\\ &= n^2-n^2-4n+10n+4-25\\ &= 6n-21\\ &= 3(2n-7)\end{aligned}$$

(2 marks)

3. Show that the following statement is true

$$(n+2)^2 - 3(n+4) \equiv (n+4)(n-3) + 4$$

$$\begin{aligned}(n+2)^2 - 3(n+4) &= (n+2)(n+2) - 3(n+4)\\ &= n^2 + 2n + 2n + 4 - 3n - 12\\ &= n^2 + n - 12 + 4\\ &= (n+4)(n-3) + 4\end{aligned}$$

(2 marks)

4. Show that the following statement is true

$$3(n+3)(n-1) - 3(1-n) \equiv (3n-3)(n+4)$$

$$\begin{aligned}3(n+3)(n-1) - 3(1-n) &= 3(n^2 + 3n - n - 3) - (3 - 3n)\\ &= 3(n^2 + 2n - 3) - 3 + 3n\\ &= 3n^2 + 6n - 9 - 3 + 3n\\ &= 3n^2 + 9n - 12\\ &= (3n-3)(n+4)\end{aligned}$$

(2 marks)

5. Prove that

$$(n+3)^2 + n(3-n) - 3(n+4)$$

is a multiple of 3 for all integer values of n.

$$\begin{aligned}(n+3)^2 + n(3-n) - 3(n+4) &= (n+3)(n+3) + (3n - n^2) - (3n + 12)\\ &= n^2 + 3n + 3n + 9 + 3n - n^2 - 3n - 12\\ &= 6n - 3\\ &= 3(2n-1)\end{aligned}$$

3 can be removed as a factor, so is divisible by 3 and therefore a multiple of 3.

(2 marks)

6. Prove algebraically that the sum of two consecutive numbers is odd.

If a number is n*, then the next number is* $n+1$*.*

The sum is therefore $n + n + 1 = 2n + 1$

By definition, $2n$ *is even, and so* $2n+1$ *must be odd.*

(2 marks)

7. Prove algebraically that the sum of the squares of two consecutive multiples of 5 is not a multiple of 10.

A multiple of 5 would be $5n$, the next consecutive multiple of 5 would be $5n+5$

$$(5n)^2 + (5n+5)^2 = 25n^2 + 25n^2 + 25n + 25n + 25$$
$$= 50n^2 + 50n + 25$$

What would the remainder be if this number were divided by 10?

$$50n^2 + 50n + 25 = 10(5n^2 + 5n + 2) + 5$$

Remainder of 5

(2 marks, 1 mark)

8. Tom says that $7x - (2x+3)(x+2)$ is always negative.

Is he correct? Explain your answer.

$$7x - (2x+3)(x+2) = 7x - (2x^2 + 4x + 4x + 6)$$
$$= 7x - 2x^2 - 8x - 6$$
$$= -2x^2 - x - 6$$
$$= -(2x^2 + x + 6)$$

$2x^2 + x + 6$ *is always positive, so multiplying by negative number means that the answer is always negative, as tom suggest.*

Change a single number in Tom's statement that would lead to a change in your conclusion. Why is this the case?

Changing the $+3$ into a -3 or $+2$ into a -2 would give a negative number when expanding the brackets, but would then be made positive when multiplying by negative 1.

(3 marks, 1 mark)

9. Show that the difference between 14^{20} and 21^{2} is a multiple of 7.

$$\begin{aligned} 14^{20} - 21^{2} &= (7 \times 2)^{20} - (7 \times 3)^{2} \\ &= 7^{20} \times 2^{20} - 7^{2} \times 3^{2} \\ &= 7(7^{19} \times 2^{20} - 7 \times 3^{2}) \end{aligned}$$

A factor of 7 can be taken out, so the answer must be divisible by 7, and therefore a multiple of 7.

(3 marks)

10. Show that $3^{60} - 25$ is not a prime number.

3^{60} *is always odd, because you are multiplying odd numbers.*
25 *is odd.*

The difference between two odd numbers, i.e. subtracting them, is always even.

$3^{60} - 25$ *is going to be even.*

All even numbers are divisible by 2*, so* $3^{60} - 25$ *is not prime.*

(2 marks)

11. Part of a 10x10 1-100 number grid is pictured below.

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

A 2x2 square of numbers is selected.
The following operation is performed:

Difference of the leading diagonal × Difference of the other diagonal

$$(23 - 12) \times (22 - 13) = 11 \times 9 = 99$$

Verify that this is also the case for a different 2x2 square of numbers on the grid.

Consider the square shown, for example:

$$(44 - 33) \times (43 - 34) = 11 \times 9 = 99$$

By generalising, prove this result for all possible 2x2 squares on the grid.

Taking the top left number as n, the other numbers can be written in terms of n.

n	$n + 1$
$n + 10$	$n + 11$

$$(n + 11 - n) \times \big(n + 10 - (n + 1)\big) = 11 \times (n + 10 - n - 1) = 11 \times 9 = 99$$

(1 mark, 3 marks)

12. The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is a re-arrangement of the general quadratic equation

$$ax^2 + bx + c = 0$$

By completing the square on the general quadratic equation, prove this result.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac}{4a^2} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac - b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{\pm\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(5 marks)