



MATHS MADE EASY

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

GCSE MATHEMATICS

H

Higher Tier

Paper 3 Calculator

Tuesday 13 June 2017

Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use

Pages	Mark
2-3	
4-5	
6-7	
8-9	
10-11	
12-13	
14-15	
16-17	
18-19	
20-21	
22-23	
24-25	
26	
TOTAL	



JUN1783003H01

IB/M/Jun17/E6

8300/3H

Answer all questions in the spaces provided

1 $\mathbf{a} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$2(-4) + 3 = -5$$
$$2(-1) + (-1) = -3$$

Circle the vector $2\mathbf{a} + \mathbf{b}$

[1 mark]

$$\begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -11 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -5 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -11 \\ -1 \end{pmatrix}$$

2 Which of these values of n makes 2.7×10^n a cube number?

Circle your answer.

[1 mark]

0

1

2

3

$$2.7 \times 10 = 27 = 3^3$$

3 Rearrange $2x = \frac{y}{w}$ to make w the subject.

Circle your answer.

[1 mark]

$$w = \frac{2y}{x}$$

$$w = \frac{2x}{y}$$

$$w = \frac{y}{2x}$$

$$w = \frac{x}{2y}$$

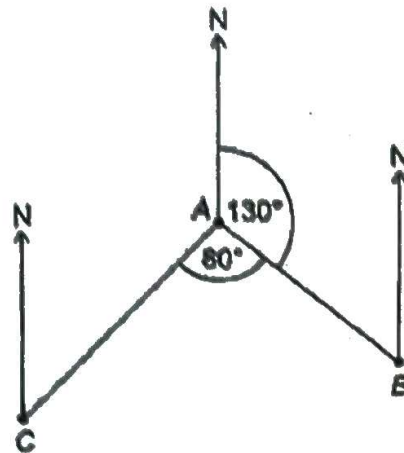
$$2xw = y$$

$$w = \frac{y}{2x}$$



3

Not drawn
accurately



$$130 + 80 = 210$$

Work out the bearing of C from A.
Circle your answer.

[1 mark]

030°

130°

150°

210°

Turn over for the next question

Turn over ►



5

A coin lands on Tails 200 times.
The relative frequency of Tails is 0.4

Work out the number of times the coin was thrown.

[2 marks]

$$\frac{200}{\text{TOTAL}} = 0.4 \Rightarrow 200 = 0.4 \times (\text{TOTAL})$$

$$\Rightarrow \frac{200}{0.4} = \text{TOTAL} = 500$$

Answer 500

6

How are the whole number solutions to A and B different?

A Solve $3 < 3x < 18$

B Solve $3 < 3x < 18$

[2 marks]

A: $3x = 3, 4, 5, \dots, 16, 17$

B: $3x = 4, 5, \dots, 16, 17, 18.$

DIFFERENCES: A HAS $3x = 3 \Rightarrow x = 1$

B HAS $3x = 18 \Rightarrow x = 6$



- 7 (a) The length of a pipe is 6 metres to the nearest metre.
Complete the error interval for the length of the pipe.

[2 marks]

Answer 5.5 m < length < 6.5 m

- 7 (b) The length of a different pipe is 4 metres to the nearest metre.
Oily says,

"The total length of the two pipes is 11 metres to the nearest metre."

Give an example to show that he could be correct.

[2 marks]

$$\begin{array}{l} \text{PIPE 1 LENGTH} = 6.3 \text{ m} \\ \text{PIPE 2 LENGTH} = 4.3 \text{ m} \end{array} \left. \vphantom{\begin{array}{l} \text{PIPE 1 LENGTH} \\ \text{PIPE 2 LENGTH} \end{array}} \right\} 6.3 + 4.3 = 10.6 \text{ m} \\ \hspace{15em} = 11 \text{ m (TO NEAREST METRE)}$$

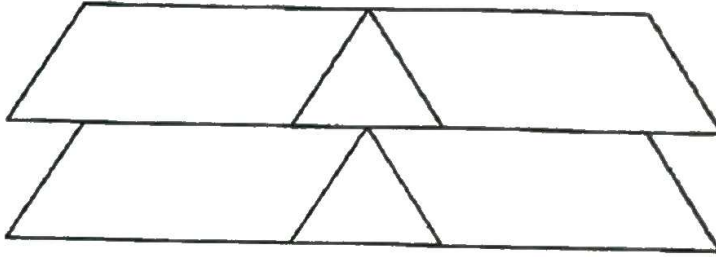
Turn over for the next question

8

Turn over ►



- 8 This shape is made from two triangles and four congruent parallelograms.



Not drawn
accurately

For each statement, tick the correct box.

- 8 (a) The triangles are equilateral.

[1 mark]

Must be true

Could be true

Must be false

- 8 (b) The triangles are congruent.

[1 mark]

Must be true

Could be true

Must be false



9 There are 720 boys and 700 girls in a school.

The probability that a boy chosen at random studies French is $\frac{2}{3}$

The probability that a girl chosen at random studies French is $\frac{3}{5}$

9 (a) Work out the number of students in the school who study French.

[3 marks]

$$\frac{2}{3} \times 720 = 480 \text{ BOYS STUDY FRENCH}$$

$$\frac{3}{5} \times 700 = 420 \text{ GIRLS STUDY FRENCH}$$

$$420 + 480 = 900$$

Answer 900

9 (b) Work out the probability that a student chosen at random from the whole school does not study French.

[2 marks]

$$(720 + 700) - 900 = 520 \text{ DON'T STUDY FRENCH}$$

$$\frac{520}{720 + 700} = \frac{26}{71}$$

Answer $\frac{26}{71}$

Turn over for the next question

7

Turn over ►

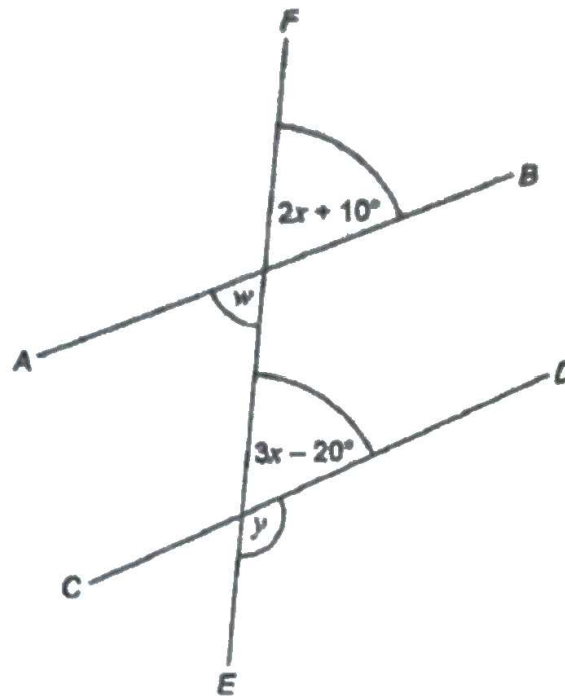


07

MATHS MADE EASY

10

AB, CD and EF are straight lines.

Not drawn
accurately

10 (a) Ava assumes that AB and CD are parallel.

What answer should she get for the size of angle y ?

[4 marks]

$$\text{(CORRESPONDING ANGLES: } 2x + 10 = 3x - 20$$

$$\Rightarrow 2x + 30 = 3x$$

$$\Rightarrow 30 = 3x - 2x = x$$

 $x = 30$. THEN,

$$y = 180 - (3x + 20) = 180 - (3 \times 30 + 20)$$

$$= 180 - 70 = 110$$

Answer 110 degrees

0 8

MATHS MADE EASY

10 (b)

In fact,

AB and CD are not parallel

angle w is 60° What effect does this have on the size of angle y ?

Tick a box.

 y is bigger y is the same y is smaller

Show working to support your answer.

[3 marks]

$$\begin{array}{r|l} \text{OPPOSITE ANGLES : } (w =) & 60 = 2x + 10 \\ -10 & 50 = 2x \\ \div 2 & 25 = x \end{array} \quad \begin{array}{l} -10 \\ \div 2 \end{array}$$

$$x = 25 \Rightarrow 3x - 20 = 3(25) - 20 = 55$$

$$\therefore y = 180 - 55 = 125 > 110$$

Turn over for the next question

Turn over ►



11

Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2

Yan has 30 litres of red paint and 9 litres of blue paint.

What is the maximum amount of purple paint he can make?

[3 marks]

TRY: USE ALL 30L OF RED PAINT. SO,

$$5 : 2 = 30 : 12, \text{ THIS WOULD REQUIRE 12L OF BLUE NOT POSSIBLE.}$$

$\xrightarrow{\times 6}$

TRY: USE ALL 9L OF BLUE PAINT. SO,

$$5 : 2 = 22.5 : 9, \text{ WHICH IS POSSIBLE (AND } \therefore \text{ MAXIMUM)}$$

$$\xrightarrow{\times 4.5} \quad \text{TOTAL} = 9 + 22.5 = 31.5$$

Answer 31.5 litres

12

$$(ar^b)^4 = 16r^{20} \text{ where } a \text{ and } b \text{ are positive integers.}$$

Work out a and b

[2 marks]

$$(ar^b)^4 = a^4 \times r^{4b} = 16r^{20}$$

$$\Rightarrow a^4 = 16, \text{ SO } a = \sqrt[4]{16} = 2$$

$$\text{ALSO, } r^{4b} = r^{20} \Rightarrow 4b = 20 \Rightarrow b = 5$$

$$a = \underline{2} \quad b = \underline{5}$$

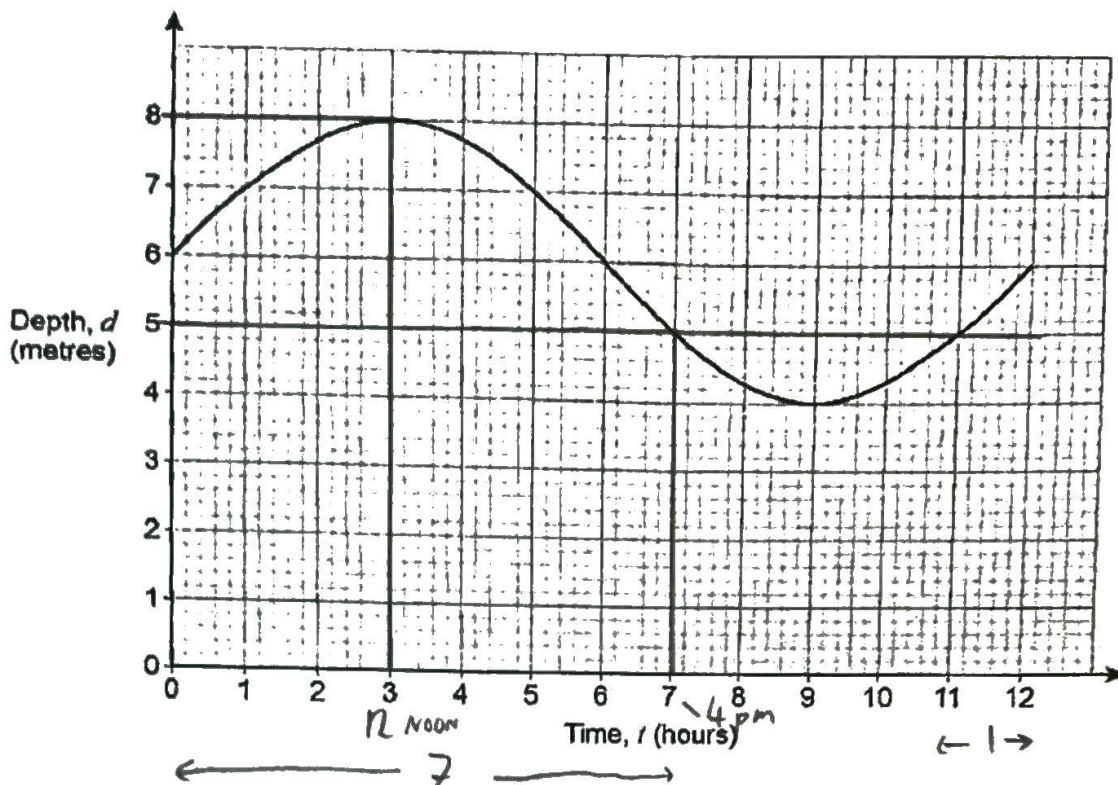


15

The graph shows the depth of water in a harbour for 12 hours.

d is the depth of water in a harbour in metres

t is the number of hours after 9 am



15 (a)

For how many of the 12 hours is the depth more than 5 metres?

[1 mark]

$$7 + 1 = 8$$

Answer

8

15 (b)

By how much does the depth change between 12 noon and 4 pm?

[1 mark]

$$8 - 5 = 3$$

Answer

3

metres



16

The value of a new car is £18 000

The value of the car decreases by

25% in the first year — 0.75 MULTIPLIER

12% in each of the next 4 years. — 0.88 MULTIPLIER

Work out the value of the car after 5 years.

[3 marks]

$$\text{AFTER 1 YEAR: } 18,000 \times 0.75 = 13,500$$

AFTER 4 MORE YEARS (SO 5 IN TOTAL):

$$13,500 \times 0.88^4 = 8095.89 \text{ (2d.p.)}$$

Answer £ 8095.89

Turn over for the next question

5

Turn over ►



1 3

MATHS MADE EASY

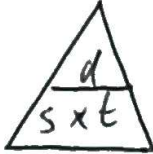
17

Liam drives his car.

He drives the first 9 miles in 9 minutes.

He then drives at an average speed of 70 miles per hour for 1 hour 36 minutes.

He finds this information about his car.



Average speed	Miles travelled per gallon
65 miles per hour or less	50
More than 65 miles per hour	40

Use the information to show that his car uses less than 3 gallons of petrol for the drive.

[5 marks]

$$9 \text{ MINUTES} = \frac{9}{60} \text{ HOURS} = 0.15 \text{ HOURS.}$$

$$\text{SPEED IN FIRST 9 MINUTES} = \frac{9}{0.15} = 60 \text{ mph} < 65$$

$$\text{GALLONS USED IN FIRST 9 MINUTES} = \frac{9}{50} = 0.18 \text{ GALLONS}$$

$$\text{SPEED} = 70 > 65, \text{ SO } 40 \text{ MILES PER GALLON.}$$

$$\begin{aligned} \text{DISTANCE TRAVELLED} &= 70 \times \left(1 \frac{36}{60}\right) \\ &= 112 \text{ MILES} \end{aligned}$$

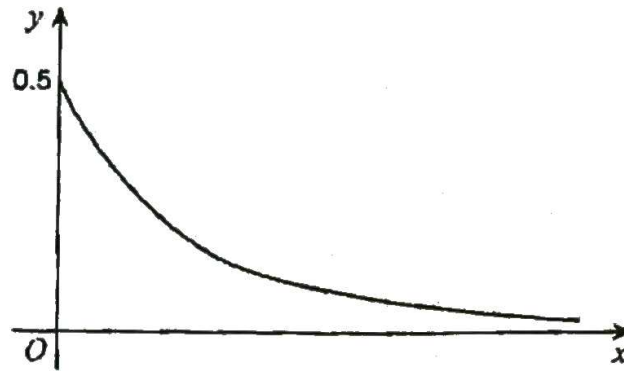
$$\text{GALLONS USED IN THIS PERIOD} = \frac{112}{40} = 2.8 \text{ GALLONS}$$

$$\text{TOTAL GALLONS USED} = 2.8 + 0.18 = \underline{2.98} < 3$$

$$\text{MILES PER GALLON} = \frac{\text{MILES}}{\text{GALLONS}} \Rightarrow \text{GALLONS} = \frac{\text{MILES}}{\text{MILES PER GALLON}}$$



- 18 Nick sketches the graph of $y = 0.5^x$ for $x > 0$



Make one criticism of his sketch.

[1 mark]

$(0.5)^0 = 1$, SO HIS y -INTERCEPT SHOULD BE 1.

Turn over for the next question

Turn over ►

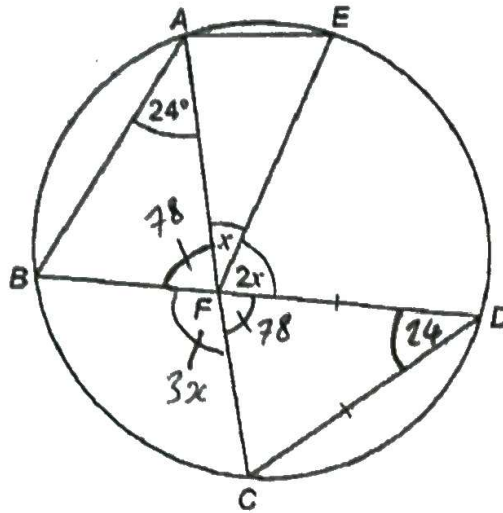


19

A, B, C, D and E are points on a circle.

BFD and AFC are straight lines.

DC = DF

Not drawn
accurately

ANGLE FDC = ANGLE BAF,
THEY ARE ANGLES IN THE
SAME SEGMENT.

Work out the size of angle x .

You must show your working which may be on the diagram.

[4 marks]

$$FCD \text{ ISOSCELES} \Rightarrow \text{ANGLE } CFD = \frac{180 - 24}{2} = 78$$

$$\text{ANGLE } AFB = \text{ANGLE } CFD, \text{ OPPOSITE ANGLES}$$

$$\text{ANGLE } BFC = \text{ANGLE } AFD = x + 2x = 3x, \text{ OPPOSITE ANGLES.}$$

ANGLES AROUND A POINT:

$$3x + 78 + x + 2x + 78 = 360$$

$$\Rightarrow 6x + 156 = 360 \Rightarrow 6x = 204 \Rightarrow x = 34$$

Answer 34 degrees

20

This sign shows when a lift is safe to use.

Total mass of people must be 450 kg or less

Ben and some other people are in the lift.

Their total mass is 525 kg to the nearest 5 kg

Ben gets out.

He has a mass of 78 kg to the nearest kg

Is the lift now safe to use?

You must show your working.

WORST CASE

[4 marks]

$$\text{MAXIMUM TOTAL MASS} < 527.5 \text{ kg}$$

$$\text{MINIMUM BEN MASS} = 77.5 \text{ kg.}$$

$$527.5 - 77.5 = 450 \text{ kg}$$

$$\therefore \text{MAXIMUM TOTAL MASS} - \text{MINIMUM BEN MASS} < 450$$

Answer

YES

Turn over for the next question

8

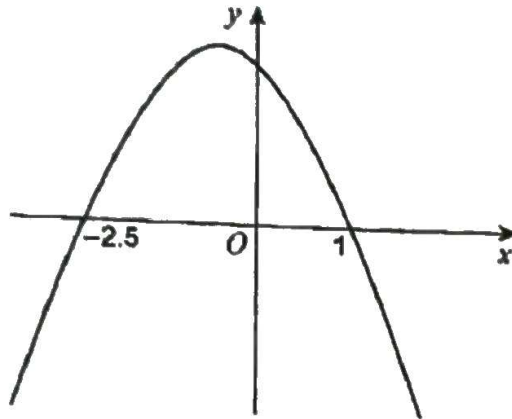
Turn over ►



17

21

Here is a sketch of $y = f(x)$ where $f(x)$ is a quadratic function.
The graph intersects the x -axis where $x = -2.5$ and $x = 1$

Not drawn
accuratelyCircle the solution of $f(x) > 0$

[1 mark]

$$x < -2.5 \text{ or } x > 1$$

$$x > -2.5 \text{ or } x > 1$$

$$-2.5 < x < 1$$

$$x > -2.5 \text{ or } x < 1$$



22

Work out an expression for the n th term of the quadratic sequence

$$U_n: \quad 2 \quad \xrightarrow{+15} \quad 17 \quad \xrightarrow{+23} \quad 40 \quad \xrightarrow{+31} \quad 71 \quad \dots$$

$\xrightarrow{+8}$ $\xrightarrow{+8}$
 $\xrightarrow{+8}$ $\xrightarrow{+8}$

Give your answer in the form $an^2 + bn + c$ where a , b and c are constants. [3 marks]

SECOND DIFFERENCE = 8 $\Rightarrow a = \frac{8}{2} = 4$

n	1	2	3	4	
U_n	2	17	40	71	n^{th} TERM FOR d_n :
$4n^2$	4	16	36	64	$3n + k$
d_n	-2	1	4	7	$n=1: d_1 = -2 = 3(1) + k$
		$+3$	$+3$	$+3$	$\Rightarrow k = -5$
					$\Rightarrow d_n = 3n - 5$

$$d_n = U_n - 4n^2$$

COMBINING BOTH PARTS OF THE n^{th} TERM FORMULA:

$$U_n = 4n^2 + 3n - 5$$

Answer $4n^2 + 3n - 5$

Turn over for the next question

4

Turn over ►

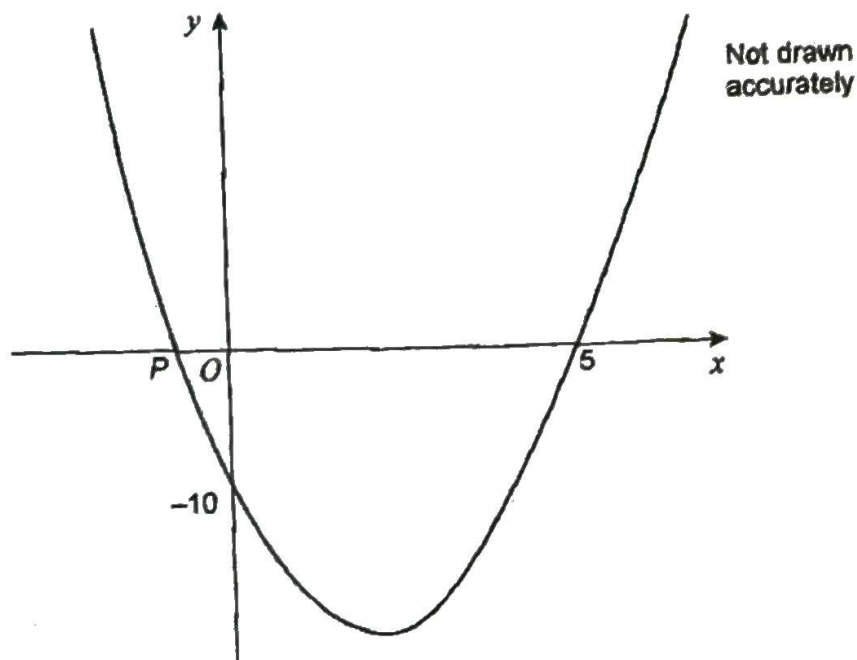


1 9

23

Here is a sketch of $y = x^2 + bx + c$

The curve intersects

the x -axis at $(5, 0)$ and point P the y -axis at $(0, -10)$ Work out the x -coordinate of the turning point of the graph.

[4 marks]

$$x = 0, \quad -10 = y = 0^2 + 0 \times b + c$$

$$\Rightarrow c = -10$$

$$x = 5, \quad 0 = y = 5^2 + 5b - 10$$

$$\Rightarrow 0 = 5b + 15 \Rightarrow 5b = -15 \Rightarrow b = -3$$

$$\text{So, } y = x^2 - 3x - 10$$

$$\text{COMPLETE THE SQUARE: } y = \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 10$$

$$\therefore x = \frac{3}{2} \text{ IS TURNING POINT CO-ORD.}$$

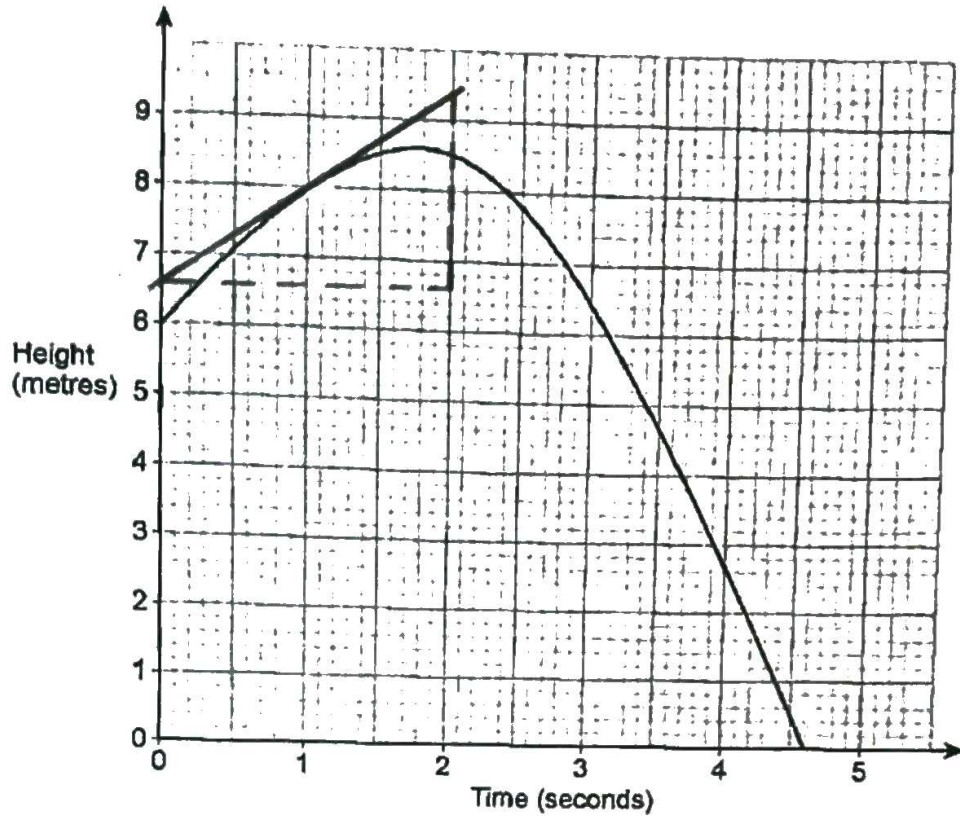
Answer

$$x = \frac{3}{2}$$



24

A ball is thrown from a point 6 metres above the ground.
The graph shows the height of the ball above the ground, in metres.



GRADIENT OF TANGENT

Estimate the speed of the ball, in m/s, after 1 second.

You must show your working.

[2 marks]

$$\text{GRADIENT} = \frac{\text{CHANGE IN } y}{\text{CHANGE IN } x} = \frac{9.4 - 6.6}{2 - 0} = \frac{2.8}{2} = 1.4$$

Answer 1.4 m/s

6

Turn over ►



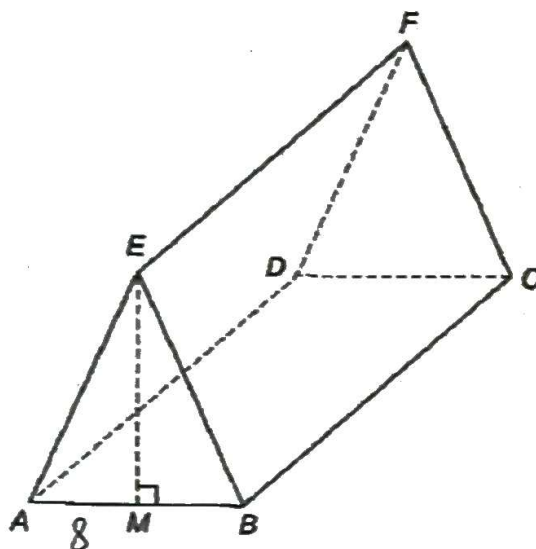
25

Rectangle $ABCD$ is the horizontal base of a triangular prism $ABCDEF$.

$$AE = BE$$

E is vertically above M , the midpoint of AB .

$$AB = 16 \text{ cm} \quad AE = 17 \text{ cm} \quad BC = 30 \text{ cm}$$



25 (a) Show that $EM = 15 \text{ cm}$

[2 marks]

$$AM = \frac{1}{2}AB = 8 \text{ cm}, \quad AE = 17 \text{ cm}$$

$$(AE)^2 = (AM)^2 + (EM)^2 \Rightarrow 17^2 = 8^2 + EM^2$$

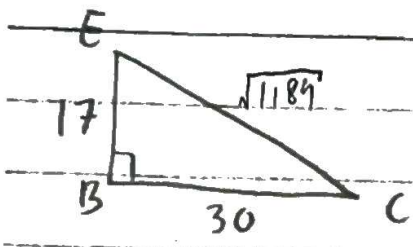
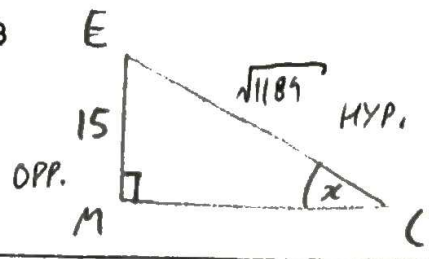
$$\Rightarrow \sqrt{17^2 - 8^2} = EM = \sqrt{225}$$

$$= 15$$



25 (b) Work out the size of angle ECM.

[4 marks]



$$(EC)^2 = 17^2 + 30^2$$

$$\Rightarrow EC = \sqrt{17^2 + 30^2}$$

$$\Rightarrow EC = \sqrt{1189} = 34.48\dots$$

SOMC AHT O A

$$\sin x = \frac{O}{H} = \frac{15}{\sqrt{1189}}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{15}{\sqrt{1189}}\right) = 25.8^\circ \text{ (3s.f.)}$$

Answer 25.8 degrees

Turn over for the next question

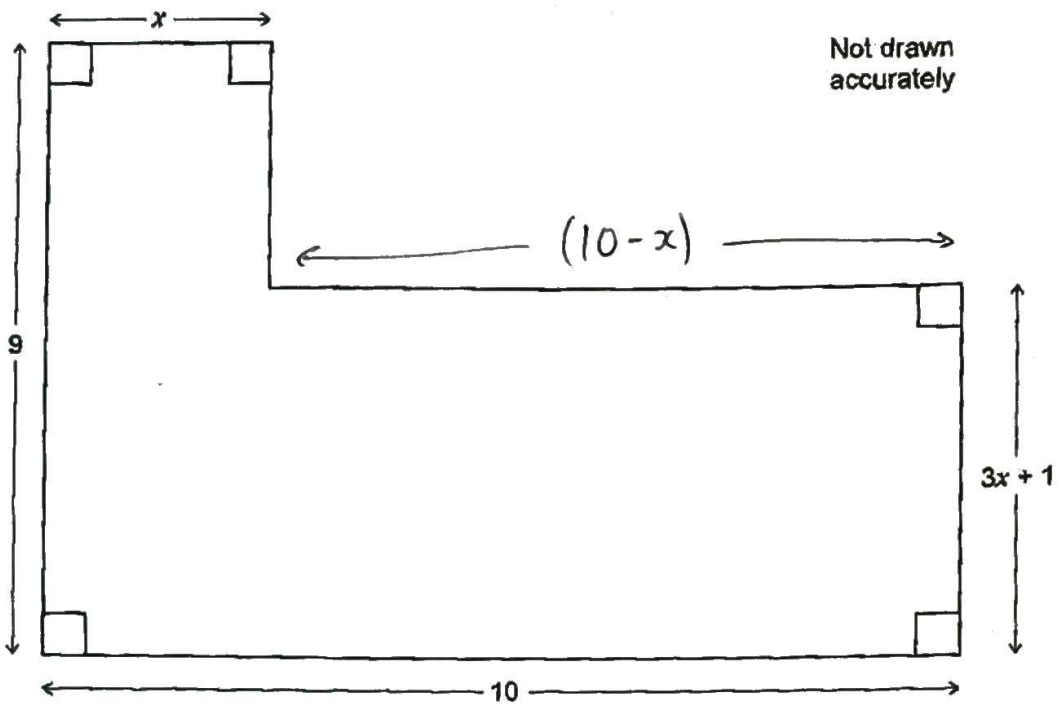
Turn over ►



26

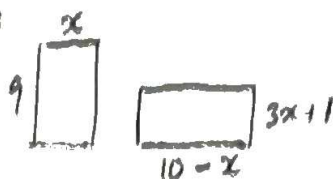
Here is an L-shape.

All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x .



[6 marks]

$$\text{AREA} = 65 = 9x + (10-x)(3x+1)$$

$$\Rightarrow 65 = 9x + (30x + 10 - 3x^2 - x)$$

$$\Rightarrow 65 = 38x + 10 - 3x^2$$

$$\Rightarrow 65 - 38x - 10 + 3x^2 = 0$$

$$\Rightarrow 3x^2 - 38x + 55 = 0$$

NEED TWO NEGATIVE NUMBERS TO ADD TO

NOW, $3 \times 55 = 165$, THEN

$$165 = (-1) \times (-165) = (-3) \times (-55) = (-5) \times (-33)$$

$$(-5) + (-33) = -38 \checkmark$$

$$\text{THEN: } 3x^2 - 38x + 55 = 0$$

$$\Rightarrow 3x^2 - 33x - 5x + 55 = 0$$

$$\Rightarrow 3x(x-11) - 5(x-11) = 0$$

$$\Rightarrow (x-11)(3x-5) = 0$$

$$\therefore x = 11 \text{ OR } x = \frac{5}{3}$$

MUST HAVE $x = \frac{5}{3}$, BECAUSE FROM THE DIAGRAM

$$x < 10.$$

Answer

$$x = \frac{5}{3}$$

Turn over for the next question

Turn over ►



27 Prove that $x^2 + x + 1$ is always positive.

[3 marks]

COMPLETE THE SQUARE:

$$\begin{aligned} x^2 + x + 1 &= \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1 \\ &= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \end{aligned}$$

$\left(x + \frac{1}{2}\right)^2$ IS A SQUARE, SO IS ALWAYS GREATER THAN OR EQUAL TO ZERO. THEREFORE,

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \geq 0 + \frac{3}{4} = \frac{3}{4},$$

SO IT MUST ALWAYS BE POSITIVE.

END OF QUESTIONS

