## GCE Examinations

## Advanced Subsidiary / Advanced Level

## Statistics

## Module S2

## Paper F

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.
Accuracy marks (A) can only be awarded when a correct method has been used.
(B) marks are independent of method marks.

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## S2 Paper F - Marking Guide

1. (a) $=\mathrm{e}^{-1.4}\left(1+1.4+\frac{1.4^{2}}{2}+\frac{1.4^{3}}{3!}\right)$

M1 A1
$=0.9463$ (4sf)
(b) $\operatorname{let} A \sim \mathrm{~B}(20,0.4)$

A1

- M1
$\mathrm{P}(Y \leq 12)=\mathrm{P}(A \geq 8) \quad$ M1
$=1-\mathrm{P}(A \leq 7) \quad$ M1
$=1-0.4159=0.5841 \quad$ A1
(7)

2. (a) frame - list of all learners she has taught
units - individual learners
B1
(b) let $X=$ no. of learners failing first 2 attempts $\therefore X \sim \mathrm{~B}\left(120, \frac{1}{20}\right)$
$\begin{array}{lll}\mathrm{H}_{0}: p=\frac{1}{20} & \mathrm{H}_{1}: p \neq \frac{1}{20} & \text { B1 }\end{array}$
Po approx. $X \approx \sim \operatorname{Po}(6) \quad$ M1
$\mathrm{P}(X \leq 1)=0.0174, \mathrm{P}(X \leq 11)=0.9799 \quad$ M1 A1
$\therefore$ C.R. is $X \leq 1$ or $X \geq 12$
(c) $0.0174+0.0201=0.0375$
A1
(9)
3. (a) mean $=16$

A1
variance $=\frac{1}{12}(28-4)^{2}=48$
M1 A1
(b) $=\mathrm{P}(13<X<19)$

M1
$=6 \times \frac{1}{24}=\frac{1}{4}$
M1 A1
(c) let $Y=$ no. within 3 cm of middle $\therefore Y \sim \mathrm{~B}\left(12, \frac{1}{4}\right)$

M1
$\mathrm{P}(Y>4)=1-\mathrm{P}(Y \leq 4)=1-0.8424=0.1576$
M1 A1
4. (a) events must occur singly, at random, at constant rate

B2
fairly valid although rate may vary through evening
B1
(b) let $X=$ no. of visitors per 10 minutes $\therefore X \sim \operatorname{Po}(5)$

M1
$\mathrm{P}(X<2)=\mathrm{P}(X \leq 1)=0.0404$
M1 A1
(c) let $Y=$ no. of visitors per 15 minutes $\therefore Y \sim \operatorname{Po}(7.5)$

M1
$\mathrm{P}(Y \geq 10)=1-\mathrm{P}(Y \leq 9)=1-0.7764=0.2236$
M1 A1
(d) $\quad$ let $A=$ no. of visitors per 3 hours $\therefore A \sim \operatorname{Po}(90)$

M1
N approx. $B \sim \mathrm{~N}(90,90)$
M1
$\mathrm{P}(A>100) \approx \mathrm{P}(B>100.5)$
M1
$=\mathrm{P}\left(Z>\frac{100.5-90}{\sqrt{90}}\right)=\mathrm{P}(Z>1.11)$
$=1-0.8665=0.1335$
A1
A1 (14)
5. (a) binomial, $n=4, p=\frac{1}{2}$
fixed no. of coins flipped, 2 outcomes, $p$ fixed
(b) $\quad H \sim \mathrm{~B}\left(4, \frac{1}{2}\right)$
$\mathrm{P}($ more heads $)=\mathrm{P}(H \geq 3)$

$$
\begin{aligned}
& =4\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^{4} \\
& =\frac{4}{16}+\frac{1}{16}=\frac{5}{16}
\end{aligned}
$$

(c) let $X=$ no. of times get more heads $\therefore X \sim \mathrm{~B}\left(5, \frac{5}{16}\right)$
$\mathrm{H}_{0}: p=\frac{5}{16} \quad \mathrm{H}_{1}: p>\frac{5}{16} \quad$ B1
$\mathrm{P}(X \geq 4)=5\left(\frac{5}{16}\right)^{4}\left(\frac{11}{16}\right)+\left(\frac{5}{16}\right)^{5}$
M1

$$
=0.0358(3 \mathrm{sf})
$$

A1
less than $5 \% \quad \therefore$ significant, evidence of higher prob.
(d) $\mathrm{P}($ head $): \mathrm{P}($ tail $)=1.5: 1=3: 2 \therefore \mathrm{P}($ head $)=\frac{3}{5}$
$\therefore H \sim \mathrm{~B}\left(4, \frac{3}{5}\right)$

$$
\begin{align*}
\mathrm{P}(H \geq 3) & =4\left(\frac{3}{5}\right)^{3}\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)^{4} & & \text { M1 A1 } \\
& =\frac{297}{625} \text { or } 0.4752(4 \mathrm{sf}) & & \text { A1 } \tag{17}
\end{align*}
$$

6. (a)

(b) $\mathrm{E}(X)=\int_{2}^{6} x \times \frac{1}{16} x \mathrm{~d} x=\frac{1}{16} \int_{2}^{6} x^{2} \mathrm{~d} x$ M1

$$
=\frac{1}{48}\left[x^{3}\right]_{2}^{6}=\frac{1}{48}(216-8)=\frac{13}{3}
$$

M1 A1
(c) $\mathrm{E}\left(X^{2}\right)=\int_{2}^{6} x^{2} \times \frac{1}{16} x \mathrm{~d} x=\frac{1}{16} \int_{2}^{6} x^{3} \mathrm{~d} x$

M1

$$
=\frac{1}{64}\left[x^{4}\right]_{2}^{6}=\frac{1}{64}(1296-16)=20
$$

$\therefore \operatorname{Var}(X)=20-\left(\frac{13}{3}\right)^{2}=\frac{11}{9}$
M1 A1
(d) $\mathrm{F}(t)=\int_{2}^{t} \frac{1}{16} x \mathrm{~d} x$

M1

$$
=\frac{1}{32}\left[x^{2}\right]_{2}^{t}=\frac{1}{32}\left(t^{2}-4\right)
$$

$\therefore \mathrm{F}(x)= \begin{cases}0, & x<2, \\ \frac{1}{32}\left(x^{2}-4\right) \\ 1, & 2 \leq x \leq 6, \\ x>6 .\end{cases}$
(e) $\quad \mathrm{F}\left(\mathrm{Q}_{1}\right)=\frac{1}{4} \quad \therefore \frac{1}{32}\left(x^{2}-4\right)=\frac{1}{4}$ M1
$\begin{array}{ll}x^{2}-4=8 ; x^{2}=12 ; x= \pm 2 \sqrt{ } 3 ; 2 \leq x \leq 6 \text { so } \mathrm{Q}_{1}=2 \sqrt{ } 3 & \text { M1 A1 } \\ \mathrm{F}\left(\mathrm{Q}_{3}\right)=\frac{3}{4} \therefore \frac{1}{32}\left(x^{2}-4\right)=\frac{3}{4} & \\ x^{2}-4=24 ; x^{2}=28 ; x= \pm 2 \sqrt{ } 7 ; 2 \leq x \leq 6 \text { so } \mathrm{Q}_{3}=2 \sqrt{ } 7 & \text { M1 } \\ \therefore \operatorname{IQR}=2 \sqrt{ } 7-2 \sqrt{ } 3=2(\sqrt{7}-\sqrt{3}) & \text { A1 }\end{array}$

Performance Record - S2 Paper F

| Question no. | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic(s) | Poisson, binomial | $\begin{aligned} & \text { sampling, } \\ & \text { Po appr. to } \\ & \text { binomial, } \\ & \text { hyp. test } \end{aligned}$ | $\begin{aligned} & \text { rect. dist., } \\ & \text { binomial } \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Poisson, } \\ \text { N approx. } \end{array}$ | binomial, hyp. test | p.d.f., mean, variance, c.d.f. IQR |  |
| Marks | 7 | 9 | 9 | 14 | 17 | 19 | 75 |
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