## GCE Examinations

## Advanced Subsidiary / Advanced Level

## Statistics

## Module S2

## Paper C

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.
Accuracy marks (A) can only be awarded when a correct method has been used.
(B) marks are independent of method marks.

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## S2 Paper C - Marking Guide

1. (a) (i) e.g. all individuals or items of relevance B1
(ii) e.g. a selection of individuals or items from a population B1
(b) (i) census - e.g. need to know requirements of all for catering B2
(ii) sample - e.g. testing is destructive, none left after census B2
(6)
2. (a) let $X=$ no. of complaints per day $\therefore X \sim \operatorname{Po}(6)$
$\mathrm{P}(X=3)=0.1512-0.0620=0.0892$
M1
M1 A1
(b) $\mathrm{P}(X \geq 10)=1-\mathrm{P}(X \leq 9)=1-0.9161=0.0839$

M1 A1
(c) let $Y=$ no. of days with 10 or more complaints $\therefore Y \sim \mathrm{~B}(6,0.0839)$

M1
$\mathrm{P}(Y \leq 1)=(0.9161)^{6}+6(0.0839)(0.9161)^{5}$

$$
=0.916(3 \mathrm{sf})
$$

M1 A1
A1
(9)
3. (a) let $X=$ no. out of 8 who take out policies $\therefore X \sim \mathrm{~B}(8,0.3)$
$\mathrm{P}(X=2)=0.5518-0.2553=0.2965$
(b) $\mathrm{P}(X>4)=1-\mathrm{P}(X \leq 4)=1-0.9420=0.0580$
(c) let $Y=$ no. out of 150 who take out policies $\therefore Y \sim \mathrm{~B}(150,0.3)$

M1
N approx. $S \sim \mathrm{~N}(45,31.5)$
M1
$\mathrm{P}(Y>50) \approx \mathrm{P}(S>50.5)$
M1
$=\mathrm{P}\left(Z>\frac{50.5-45}{\sqrt{31.5}}\right)=\mathrm{P}(Z>0.98) \quad$ A1
$=1-0.8365=0.1635 \quad \mathrm{~A} 1$
(10)
4. (a) let $X=$ no. of tries per match $\therefore X \sim \operatorname{Po}(0.4) \quad$ M1

$$
\begin{aligned}
\mathrm{P}(X \geq 2) & =1-\mathrm{P}(X \leq 1) & & \mathrm{M} 1 \\
& =1-\mathrm{e}^{-0.4}(1+0.4) & & \text { M1 A1 } \\
& =1-0.9384=0.0616(3 \mathrm{sf}) & & \text { A1 }
\end{aligned}
$$

(b) let $Y=$ no. of tries per 5 matches $\therefore Y \sim \operatorname{Po}(2)$

M1
$\mathrm{H}_{0}: \lambda=2 \quad \mathrm{H}_{1}: \lambda>2$
$\mathrm{P}(Y \geq 6)=1-\mathrm{P}(Y \leq 5)=1-0.9834=0.0166$
B1
less than $5 \% \quad \therefore$ significant, evidence of increase
5. (a) $\mathrm{P}(X<2)=\mathrm{F}(2)=\frac{1}{432} \times 4 \times(4-32+72)=\frac{11}{27}$
(b) $\quad \mathrm{F}(x)=\frac{1}{432}\left(x^{4}-16 x^{3}+72 x^{2}\right)$ M1
$\mathrm{f}(x)=\mathrm{F}^{\prime}(x)=\frac{1}{432}\left(4 x^{3}-48 x^{2}+144 x\right)$ M1 A1
$\therefore \mathrm{f}(x)=\left\{\begin{array}{ll}\frac{1}{108}\left(x^{3}-12 x^{2}+36 x\right), & 0 \leq x \leq 6, \\ 0, & \text { otherwise } .\end{array} \quad\left[\right.\right.$ or $\left.\frac{1}{108} x(x-6)^{2}\right]$
(c) $\mathrm{f}^{\prime}(x)=\frac{1}{108}\left(3 x^{2}-24 x+36\right)$

## M1

for S.P. $=0$ giving $x^{2}-8 x+12=0$
M1 A1
$\therefore(x-6)(x-2)=0$ so $x=2$ or 6
some justification, e.g. + ve cubic $/ \mathrm{f}(x)=0$ at 0 and $6 \therefore$ mode $=2$
M1
M1 A1
(d) median higher as $\mathrm{P}(X<2)$ is less than $\frac{1}{2}$

B1
6. (a) fixed no. of eggs, eggs either broken or not, prob. of each egg being broken is same (assuming no accident breaking group together)
(b) let $X=$ no. of eggs broken in delivery $\therefore X \sim \mathrm{~B}(120,0.008)$
$\mathrm{P}(X \leq 1)=(0.992)^{120}+120(0.008)(0.992)^{119}$
M1 A1 $=0.7505$ (4sf)

A1
(c) $n$ large, $p$ small

B1
(d) $X \approx \sim \operatorname{Po}(0.96)$

M1
$\mathrm{P}(X \leq 1) \approx \mathrm{e}^{-0.96}(1+0.96)$
M1 A1 $=0.7505(4 \mathrm{sf})$
same value to 4sf, very good approx. for these parameters
B1
7.
(a) 6.5

A1
(b) $2.4 \times \frac{1}{9}=\frac{4}{15}$ or $0.2667(4 \mathrm{sf})$

M1 A1
(c) $=\mathrm{P}(3<X<7)=4 \times \frac{1}{9}=\frac{4}{9}$ or $0.4444(4 \mathrm{sf})$

M1 A1
(d) $\mathrm{f}(y)=\frac{1}{b-a}$

B1
$\mathrm{E}\left(Y^{2}\right)=\int_{a}^{b} \frac{1}{b-a} y^{2} \mathrm{~d} y$
M1
$=\frac{1}{b-a}\left[\frac{1}{3} y^{3}\right]_{a}^{b}$
$=\frac{b^{3}-a^{3}}{3(b-a)}$
A1

$$
=\frac{1}{3}\left(b^{2}+a b+a^{2}\right)
$$

(e) $\operatorname{Var}(Y)=\mathrm{E}\left(Y^{2}\right)-[\mathrm{E}(Y)]^{2}$

M1
$\begin{array}{lrl}=\frac{1}{3}\left(b^{2}+a b+a^{2}\right)-\frac{1}{4}\left(a^{2}+2 a b+b^{2}\right) & & \text { M1 } \\ =\frac{1}{12}\left(4 b^{2}+4 a b+4 a^{2}-3 a^{2}-6 a b-3 b^{2}\right) & & \text { M1 } \\ =\frac{1}{12}\left(b^{2}-2 a b+a^{2}\right)=\frac{1}{12}(b-a)^{2} & & \text { A1 }\end{array}$

Performance Record - S2 Paper C

| Question no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic(s) | sampling | $\begin{array}{\|l} \hline \begin{array}{l} \text { Poisson, } \\ \text { binomial } \end{array} \\ \hline \end{array}$ | binomial, N approx. | $\begin{array}{\|l\|} \hline \text { Poisson, } \\ \text { hyp. test } \end{array}$ | $\begin{array}{\|l\|l} \hline \text { c.d.f., } \\ \text { p.d.f. } \\ \text { mode, } \\ \text { mode. } \\ \text { median } \end{array}$ | binomial, Po appr. to binomial | rect. dist., deriving varianc |  |
| Marks | 6 | 9 | 10 | 10 | 13 | 13 | 14 | 75 |
| Student |  |  |  |  |  |  |  |  |
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