## GCE Examinations

## Advanced Subsidiary / Advanced Level

## Statistics

## Module S2

## Paper B

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.
Accuracy marks (A) can only be awarded when a correct method has been used.
(B) marks are independent of method marks.

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## S2 Paper B - Marking Guide

1. (a) e.g. list of all the sampling units B1
(b) (i) frame - list of cars serviced at garage B1
units - individual cars B1
(ii) frame - list of people involved in trial B1 units - individual people

B1
(5)
2. (a) Poisson (with $\lambda=4.2$ )
(b) (i) e.g. may be more or less species that like nuts

B1
(ii) e.g. will last longer so may get more species visiting B1
(c) let $X=$ no. of species that visit $\therefore X \sim \operatorname{Po}(4.2)$

$$
\mathrm{P}(X=6)=\frac{\mathrm{e}^{-4.2} \times 4.2^{6}}{6!}=0.1143(4 \mathrm{sf})
$$

(d) $\mathrm{P}(X>2)=1-\mathrm{P}(X \leq 2)$

M1

$$
\begin{aligned}
& =1-\mathrm{e}^{-4.2}\left(1+4.2+\frac{4.2^{2}}{2}\right) \\
& =1-0.2102=0.7898(4 \mathrm{sf})
\end{aligned}
$$

M1 A1
A1 (9)
3. (a) $1.6 \times \frac{1}{20}=0.08$

M1 A1
(b) mean $=10$
variance $=\frac{1}{12}(20-0)^{2}=\frac{100}{3}$
A1
M1 A1
(c) $\quad=\mathrm{P}(X$ in middle 4 cm$) \times \mathrm{P}(Y$ in middle 4 cm$)$

M1
$=\left(4 \times \frac{1}{20}\right) \times\left(4 \times \frac{1}{16}\right)$
M1 A1
$=\frac{1}{5} \times \frac{1}{4}=\frac{1}{20}$ A1
(d) $\quad=1-[\mathrm{P}(X$ in middle 16 cm$) \times \mathrm{P}(Y$ in middle 12 cm$)]$

M1 A1
$=1-\left[\left(16 \times \frac{1}{20}\right) \times\left(12 \times \frac{1}{16}\right)\right]$
M1
$=1-\left(\frac{4}{5} \times \frac{3}{4}\right)=1-\frac{3}{5}=\frac{2}{5}$
A1
4. (a) let $X=$ no. of failures per hour $\therefore X \sim \operatorname{Po}(3)$
$\mathrm{P}(X=0)=0.0498$
(b) let $Y=$ no. of failures per half-hour $\therefore Y \sim \operatorname{Po}(1.5)$
$\mathrm{P}(Y>4)=1-\mathrm{P}(Y \leq 4)=1-0.9814=0.0186$
(c) let $F=$ no. of failures per $24 \mathrm{hrs} \therefore F \sim \operatorname{Po}(72)$

N approx. $G \sim \mathrm{~N}(72,72)$
$\mathrm{P}(F<60) \approx \mathrm{P}(G<59.5)$

$$
=\mathrm{P}\left(Z<\frac{59.5-72}{\sqrt{72}}\right)=\mathrm{P}(Z<-1.47)
$$

$$
=1-0.9292=0.0708
$$

(d) $\mathrm{P}(F=72) \approx \mathrm{P}(71.5<G<72.5)$

|  | $=\mathrm{P}\left(Z<\frac{72.5-72}{\sqrt{72}}\right)-\mathrm{P}\left(Z<\frac{71.5-72}{\sqrt{72}}\right)$ |  | M1 |
| :--- | :--- | ---: | :--- |
|  | $=\mathrm{P}(Z<0.06)-\mathrm{P}(Z<-0.06)$ |  | A1 |
|  | $=0.5239-0.4761=0.0478$ |  | A1 |

5. (a) let $X=$ no. of dice showing $6 \therefore X \sim \mathrm{~B}\left(6, \frac{1}{6}\right)$

$$
\mathrm{P}(X=0)=\left(\frac{5}{6}\right)^{6}=0.3349(4 \mathrm{sf})
$$

(b) $\mathrm{P}(X>1)=1-\mathrm{P}(X \leq 1)$

$$
\begin{aligned}
& =1-\left[\left(\frac{5}{6}\right)^{6}+6\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{5}\right] \\
& =1-0.7368=0.2632(4 \mathrm{sf})
\end{aligned}
$$

(c) let $Y=$ no. of dice showing odd $\therefore Y \sim \mathrm{~B}\left(6, \frac{1}{2}\right)$
$\mathrm{P}(Y=3)=0.6563-0.3438=0.3125$
(d) let $S=$ no. of times it shows a $6 \therefore S \sim \mathrm{~B}\left(8, \frac{1}{6}\right)$

M1
$\mathrm{H}_{0}: p=\frac{1}{6} \quad \mathrm{H}_{1}: p>\frac{1}{6}$
$\mathrm{P}(S \geq 3)=1-\mathrm{P}(S \leq 2)$
$=1-\left[\left(\frac{5}{6}\right)^{8}+8\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{7}+\frac{8 \times 7}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{6}\right]$
$=1-0.8652=0.1348$
M1 A1
A1
A1
(17)
6. (a)

(b) 2

B4
(c) 0 to 2: $\mathrm{F}(t)=\int_{0}^{t} \frac{1}{6} x \mathrm{~d} x$ A1 M1

$$
=\left[\frac{1}{12} x^{2}\right]_{0}^{t}=\frac{1}{12} t^{2}
$$

M1 A1
2 to 6: $\mathrm{F}(t)=\frac{1}{2} \times 2 \times \frac{1}{3}+\int_{2}^{t} \frac{1}{12}(6-x) \mathrm{d} x$
M1
$=\frac{1}{3}+\frac{1}{12}\left[6 x-\frac{1}{2} x^{2}\right]_{2}^{t}$
M1 A1
$=\frac{1}{3}+\frac{1}{12}\left[\left(6 t-\frac{1}{2} t^{2}\right)-(12-2)\right]$
M1
$=\frac{1}{24}\left(12 t-t^{2}-12\right)$
A1
$\therefore \mathrm{F}(x)= \begin{cases}0, & x<0, \\ \frac{1}{12} x^{2} & 0 \leq x \leq 2, \\ \frac{1}{24}\left(12 x-x^{2}-12\right) & 2 \leq x \leq 6, \\ 1, & x>6 .\end{cases}$


Performance Record - S2 Paper B

| Question no. | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic(s) | sampling | Poisson | rect. dist. | Poisson, <br> Napprox. | binomial, <br> hyp. test | p.d.f., <br> mode, <br> c.d.f., <br> median |  |
| Marks |  |  |  |  |  |  |  |
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| Student |  |  |  |  |  |  |  |
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