## GCE Examinations

## Mechanics Module M3

## Advanced Subsidiary / Advanced Level

## Paper E

Time: 1 hour 30 minutes

## Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.
Mathematical and statistical formulae and tables are available.
This paper has 7 questions.
When a numerical value of $g$ is required, use $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.

## Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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1. The velocity, $\mathbf{v} \mathrm{cm} \mathrm{s}^{-1}$, at time $t$ seconds, of a radio-controlled toy is modelled by the formula

$$
\mathbf{v}=\mathrm{e}^{2 t} \mathbf{i}+2 t \mathbf{j},
$$

where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the acceleration of the toy in terms of $t$.
(2 marks)
(b) Find, correct to 2 significant figures, the time at which the acceleration of the toy is parallel to the vector $(4 \mathbf{i}+\mathbf{j})$.
(4 marks)
(c) Explain why this model is unlikely to be realistic for large values of $t$.
(1 mark)
2. A particle $P$ of mass 0.4 kg is moving in a straight line through a fixed point $O$.

At time $t$ seconds after it passes through $O$, the distance $O P$ is $x$ metres and the resultant force acting on $P$ is of magnitude $\left(5+4 \mathrm{e}^{-x}\right) \mathrm{N}$ in the direction $O P$.

When $x=1, P$ is at the point $A$.
(a) Find, correct to 3 significant figures, the work done in moving $P$ from $O$ to $A$.
(4 marks)
Given that $P$ passes through $O$ with speed $2 \mathrm{~m} \mathrm{~s}^{-1}$,
(b) find, correct to 3 significant figures, the speed of $P$ as it passes through $A$.
3.


Fig. 1
A popular racket game involves a tennis ball of mass 0.1 kg which is attached to one end of a light inextensible string. The other end of the string is attached to the top of a fixed rigid pole.

A boy strikes the ball such that it moves in a horizontal circle with angular speed $4 \mathrm{rad} \mathrm{s}^{-1}$ and the string makes an angle of $60^{\circ}$ with the downward vertical as shown in Figure 1.
(a) Find the tension in the string.
(b) Find the length of the string.
4. A particle moves with simple harmonic motion along a straight line.

When the particle is 3 cm from its centre of motion it has a speed of $8 \mathrm{~cm} \mathrm{~s}^{-1}$ and an acceleration of magnitude $12 \mathrm{~cm} \mathrm{~s}^{-2}$.
(a) Show that the period of the motion is $\pi$ seconds.
(4 marks)
(b) Find the amplitude of the motion.
(c) Hence, find the greatest speed of the particle.
(2 marks)
5. A physics student is set the task of finding the mass of an object without using a set of scales. She decides to use a light elastic string of natural length 2 m and modulus of elasticity 280 N attached to two points $A$ and $B$ which are on the same horizontal level and 2.4 m apart.


Fig. 2
She attaches the object to the midpoint of the string so that it hangs in equilibrium 0.35 m below $A B$ as shown in Figure 2.
(a) Explain why it is reasonable to assume that the tensions in each half of the string are equal.
(b) Find the mass of the object.
(c) Find the elastic potential energy of the string when the object is suspended from it.
(2 marks)
6.


Fig. 3
Figure 3 shows part of the curve $y=x^{2}+1$. The shaded region enclosed by the curve, the coordinate axes and the line $x=1$ is rotated through $360^{\circ}$ about the $x$-axis.
(a) Find the coordinates of the centre of mass of the solid obtained.
(10 marks)
The solid is suspended from a point on its larger circular rim and hangs in equilibrium.
(b) Find, correct to the nearest degree, the acute angle which the plane surfaces of the solid make with the vertical.
(3 marks)
7. A particle of mass 0.5 kg is hanging vertically at one end of a light inextensible string of length 0.6 m . The other end of the string is attached to a fixed point.

The particle is given an initial horizontal speed of $u \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Show that the particle will perform complete circles if $u \geq \sqrt{3 g}$.

Given that $u=5$,
(b) find, correct to the nearest degree, the angle through which the string turns before it becomes slack,
(c) find, correct to the nearest centimetre, the greatest height the particle reaches above its position when the string becomes slack.

## END

