GCE Examinations

Further Pure Mathematics Module FP3

Advanced Subsidiary / Advanced Level

Paper H

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Solomon Press

Written by Rosemary Smith & Shaun Armstrong

1. Given that

$$t_{n+1} = t_n - 4 \quad \text{for} \quad n \geq 1, \quad t_1 = 3,$$

prove by induction that $t_n = 7 - 4n$ for all integers n, $n \geq 1$. **(5 marks)**

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2. *(a)* On the same Argand diagram sketch the locus of the points defined by the equations

(i) $z + z^* = 2$,

(ii) $\arg\left(\dfrac{z-2}{z+2}\right) = \dfrac{\pi}{4}$, where $\text{Im}\,(z) \geq 0$. **(6 marks)**

The region R of the complex z-plane is defined by the inequalities

$$z + z^* \leq 2, \quad \arg\left(\frac{z-2}{z+2}\right) \geq \frac{\pi}{4} \quad \text{and} \quad \text{Im}\,(z) \geq 0.$$

(b) Shade the region R on the Argand diagram. **(2 marks)**

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3. The points A, B and C with coordinates (x_{-1}, y_{-1}), (x_0, y_0) and (x_1, y_1) respectively lie on the curve $y = \text{f}(x)$ where $x_1 - x_0 = x_0 - x_{-1} = h$ and $y_n = \text{f}(x_n)$.

(a) By drawing a sketch, or otherwise, show that

$$\text{f}'(x_0) \approx \frac{\text{f}(x_0 + h) - \text{f}(x_0 - h)}{2h}.$$ **(3 marks)**

Given that

$$\text{f}'(x) = \sqrt{2x + \text{f}(x)}, \quad \text{f}(0) = 1 \text{ and } \text{f}(0.2) = 1.25,$$

(b) use two applications of the approximation in *(a)* with a step length of 0.2 to find an estimate for f(0.6).

(5 marks)

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4. The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively such that

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = \mathbf{i} + q\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \mathbf{c} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k},$$

where q is a constant and $q \neq 2$.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$, giving your answer in terms of q. **(5 marks)**

(b) Hence show that the vector $\mathbf{n} = 4\mathbf{i} - \mathbf{k}$ is perpendicular to the plane Π containing A, B and C for all real values of q.
(2 marks)

(c) Find an equation of the plane Π, giving your answer in the form $\mathbf{r.n} = p$. **(2 marks)**

Given that $q = -1$,

(d) find the volume of the tetrahedron $OABC$. **(3 marks)**

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5. *(a)* Use De Moivre's theorem to show that

$$\cos 5\theta \equiv \cos \theta(16\cos^4\theta - 20\cos^2\theta + 5).$$ **(6 marks)**

(b) By solving the equation $\cos 5\theta = 0$, deduce that

$$\cos^2\left(\frac{3\pi}{10}\right) = \frac{5-\sqrt{5}}{8}.$$ **(7 marks)**

Turn over

6. *(a)* Find the first three derivatives of $\ln\left(\dfrac{1+x}{1-2x}\right)$. **(6 marks)**

(b) Hence, or otherwise, find the expansion of $\ln\left(\dfrac{1+x}{1-2x}\right)$ in ascending powers of x up to and including the term in x^3.
(4 marks)

(c) State the values of x for which this expansion is valid. **(1 mark)**

(d) Use this expansion to find an approximate value for $\ln\frac{4}{3}$, giving your answer to 3 decimal places.
(3 marks)

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7. $$\mathbf{A} = \begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 6 & 5 & 2 \\ -1 & 0 & -2 \\ 0 & 0 & 5 \end{pmatrix} \quad \text{and}$$

$$(\mathbf{B} - 2\mathbf{I})\mathbf{A} = 3\mathbf{I} \qquad \text{(i)}$$

where a, b and c are constants and $\mathbf{I}$ is the 3×3 identity matrix.

(a) Find the values of a, b and c. **(6 marks)**

(b) Using equation (i), or otherwise, find $\mathbf{A}^{-1}$, showing your working clearly. **(2 marks)**

The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$.

(c) Find an equation satisfied by all the points which remain invariant under T.
(4 marks)

T maps the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ onto the vector $\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$.

(d) Find the values of p, q and r. **(3 marks)**

END