

GCE Examinations

Further Pure Mathematics

Module FP3

Advanced Subsidiary / Advanced Level

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1.
$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ 1 & 2 & -1 \\ 2 & k & 0 \end{pmatrix}.$$

Find the value of the constant k for which \mathbf{A} is a singular matrix.

(3 marks)

2. Solve the equation

$$z^3 = -4 + 4\sqrt{3}i,$$

giving your answers in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $0 \leq \theta < 2\pi$.

(6 marks)

3. Prove by induction that $n(n^2 + 5)$ is divisible by 6 for all positive integers n .

(7 marks)

4. The point P represents the complex number z in an Argand diagram.

Given that

$$|z - 1 + 2i| = 3,$$

(a) sketch the locus of P in an Argand diagram.

(3 marks)

T , U and V are transformations from the z -plane to the w -plane where

$$T: w = 4z,$$

$$U: w = z + 5 - i,$$

$$V: w = ze^{\frac{i\pi}{2}}.$$

(b) Describe exactly the locus of the image of P under each of these transformations.

(6 marks)

5. (a) By finding the first four derivatives of $f(x) = \cos x$, find the Taylor series expansion of $f(x)$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^3$.
(5 marks)
- (b) Use this expansion to find an estimate of $\cos \frac{\pi}{4}$, giving your answer to 4 decimal places.
(3 marks)
- (c) Find the percentage error in your answer to part (b), giving your answer to 2 significant figures.
(2 marks)
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6. Given that y satisfies the differential equation

$$\frac{d^2 y}{dx^2} = x^2 + xy - y^2, \quad y = \frac{1}{2} \text{ and } \frac{dy}{dx} = -1 \text{ at } x = 0,$$

- (a) use the Taylor series method to obtain a series for y in ascending powers of x up to and including the term in x^3 .
(6 marks)
- (b) Use your series to estimate the value of y at $x = -0.1$
(1 mark)
- (c) Use the approximation $\left(\frac{d^2 y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ with a step length of 0.1 and your answer to part (b) to estimate the value of y when $x = 0.1$
(3 marks)
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Turn over

7. Referred to a fixed origin, the straight lines l_1 , l_2 and l_3 have equations

$$l_1 : \mathbf{r} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}),$$

$$l_2 : \mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}),$$

$$l_3 : \mathbf{r} = \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k}).$$

The acute angle between l_1 and l_2 is θ .

- (a) Find the exact value of $\sin \theta$. (5 marks)

The plane Π contains the lines l_1 and l_2 .

- (b) Find an equation of Π , giving your answer in the form $ax + by + cz + d = 0$. (4 marks)

- (c) Show that the line l_3 lies on the plane Π . (4 marks)
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8. (a) \mathbf{A} and \mathbf{B} are non-singular square matrices. Prove that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. (4 marks)

The transformations $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ are defined by

$$S : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y - x \\ 2x + y \end{pmatrix} \quad \text{and} \quad T : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3x \\ x + y \end{pmatrix}.$$

- (b) Show that S represents a linear transformation. (7 marks)

- (c) Using your result in (a), or otherwise, find the matrix that represents the transformation $(ST)^{-1}$.

(6 marks)

END