

GCE Examinations
Advanced Subsidiary / Advanced Level
Further Pure Mathematics
Module FP3

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP3 Paper E – Marking Guide

1. (a) $|z - (-1 + i)| = 2$ or $|z + 1 - i| = 2$ M1 A1
 (b) $y = 0 \therefore |x + 1 - i| = 2$ M1
 $(x + 1)^2 + 1 = 4$ A1
 giving $x = -1 \pm \sqrt{3}$ A1 (5)

2. assume true for $n = k \therefore 2^k > 2k$

$$\therefore 2^{k+1} > 2 \times 2k \quad \text{M1 A1}$$

$$\therefore 2^{k+1} > 2k + 2k$$

$$\therefore 2^{k+1} > 2k + 2 \text{ as } 2k > 2 \text{ for } k \geq 3 \quad \text{M1}$$

$$\therefore 2^{k+1} > 2(k+1) \quad \text{A1}$$

$$\therefore \text{true for } n = k+1 \text{ if true for } n = k$$

if $n = 3$, $2^3 = 8$, $2 \times 3 = 6 \therefore 2^3 > 2 \times 3$ so true for $n = 3$ B1

$$\therefore \text{by induction true for integers } n, n \geq 3 \quad \text{A1} \quad \textcolor{red}{(6)}$$

- $$\begin{aligned} 3. \quad (a) \quad \ln(1 + 2x) - 2x e^{-x} &= 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 + \dots - 2x(1 - x + \frac{1}{2!}x^2 + \dots) & M1 A1 \\ &= 2x - 2x^2 + \frac{8}{3}x^3 - 2x + 2x^2 - x^3 + \dots & M1 \\ \therefore \ln(1 + 2x) &\approx \frac{5}{3}x^3 \text{ so } A = \frac{5}{3} & A1 \end{aligned}$$

$$(b) \quad \lim_{x \rightarrow 0} \left(\frac{\ln(1+2x) - 2xe^{-x}}{x^3} \right) = \lim_{x \rightarrow 0} \left(\frac{\frac{5}{3}x^3 + kx^4 + \dots}{x^3} \right) = \frac{5}{3} \quad \text{M1 A1} \quad (6)$$

4. (a)
$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & -1 \\ -3 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \therefore \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$
 is eigenvector, eigenvalue = 1 M1 A1

$$(b) \begin{vmatrix} 2-\lambda & -1 & 1 \\ 0 & 1-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = 0 \quad M1$$

$$(2-\lambda)[(1-\lambda)^2 + 3] + 1(0-3) + 1[0 + 3(1-\lambda)] = 0 \quad A1$$

$$(2-\lambda)(1-\lambda)^2 + 6 - 3\lambda - 3 + 3 - 3\lambda = 0 \quad M1$$

$$(2 - \lambda)(1 - \lambda)^2 + 6(1 - \lambda) = 0$$

$$(1 - \lambda)[(2 - \lambda)(1 - \lambda) + 6] = 0 \quad A1$$

$$\therefore \lambda = 1 \text{ or } \lambda^2 - 3\lambda + 8 = 0$$

$$“b^2 - 4ac” = 9 - 32 = -23 (< 0) \Rightarrow \text{no real roots}$$

∴ only 1 real eigenvalue

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5.	(a)	$\operatorname{Im}(z) = 2 \therefore y = 2$ $u + iv = (x + 2i)^2 = x^2 + 4xi - 4$ $u = x^2 - 4, v = 4x$ $x = \frac{v}{4}$ $\therefore u = \frac{v^2}{16} - 4 \text{ or } v^2 = 16(u + 4) \text{ which is a parabola}$	M1 A1 M1 M1 A1
	(b)	$\arg w = \arg(z^2) = 2 \arg z \therefore \arg w = \frac{\pi}{2}$	M1 A1
	(c)	$\arg w = \frac{\pi}{2} \Rightarrow u = 0, v \geq 0$ $\therefore v^2 = 16(0 + 4) = 64, v \geq 0 \therefore v = 8$ $P \text{ represents } 0 + 8i$	M1 M1 A1 A1 (11)

6.	(a)	(i) $\frac{d^2y}{dx^2} = 2x + \frac{dy}{dx} \cos x - y \sin x$ $\frac{d^3y}{dx^3} = 2 + \frac{d^2y}{dx^2} \cos x - \frac{dy}{dx} \sin x - \frac{dy}{dx} \sin x - y \cos x$	M1 A1 M1 A2
		(ii) $y_0 = 1, \left(\frac{dy}{dx}\right)_0 = 1, \left(\frac{d^2y}{dx^2}\right)_0 = 1, \left(\frac{d^3y}{dx^3}\right)_0 = 2$ $\therefore y = 1 + x + \frac{1}{2!}x^2 + 2\left(\frac{1}{3!}\right)x^3 + \dots = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$	A1 M1 A1
		(iii) $x = -0.1, y \approx 0.904666\dots = 0.905 \text{ (3sf)}$	M1 A1
	(b)	$\frac{y_1 - y_{-1}}{0.2} = x_0^2 + y_0 \cos x_0$ $y_1 = 0.2(x_0^2 + y_0 \cos x_0) + y_{-1}$ $x_{-1} = -0.1, x_0 = 0, x_1 = 0.1; y_{-1} = 0.904666\dots, y_0 = 1, y_1 = ?$ $y_1 = 0.2(0 + 1) + 0.904666\dots = 1.104666\dots = 1.10 \text{ (3sf)}$	M1 A1 A1 (13)

7. (a) $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{AC} = -\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ M1 A1
- $$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 5 \\ -1 & -2 & -4 \end{vmatrix}$$
- $$= \mathbf{i}(-4 + 10) - \mathbf{j}(-8 + 5) + \mathbf{k}(-4 + 1) = 6\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$$
- M1 A1
- $$\mathbf{n} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$
- eqn. of plane is $\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} + \mathbf{j}) \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 2 + 1 + 0 = 3$ M1 A1
- $$\mathbf{r} \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$$
- (b) $\mathbf{r} = -2\mathbf{i} - 5\mathbf{j} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ A1
- (c) at E $[(-2 + 2\lambda)\mathbf{i} + (-5 + \lambda)\mathbf{j} - \lambda\mathbf{k}] \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k}) = 3$ M1
 $-4 + 4\lambda - 5 + \lambda + \lambda = 3$ giving $\lambda = 2$ M1 A1
 $\therefore E$ is $(2, -3, -2)$ A1
- (d) E is midpoint of DF ; $D(-2, -5, 0)$ $E(2, -3, -2)$ $\therefore F(6, -1, -4)$ M1 A1 (13)
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8. (a) $\det \mathbf{M} = 2(0 - 2) - 1(0 - 2) - 1(0 - 6) = -4 + 2 + 6 = 4$ M1 A1
- matrix of cofactors: $\begin{pmatrix} -2 & 2 & -6 \\ -2 & 2 & -2 \\ 4 & -2 & 6 \end{pmatrix}$ M1 A1
- $$\therefore \mathbf{M}^{-1} = \frac{1}{4} \begin{pmatrix} -2 & -2 & 4 \\ 2 & 2 & -2 \\ -6 & -2 & 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix}$$
- M1 A1
- (b) $\frac{x-1}{3} = \frac{y+1}{-3} = \frac{z}{4} = t$
- $$\therefore x = 3t + 1, y = -3t - 1, z = 4t$$
- M1 A1
- $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{pmatrix} \begin{pmatrix} 3t+1 \\ -3t-1 \\ 4t \end{pmatrix}$$
- $$= \frac{1}{2} \begin{pmatrix} -3t-1+3t+1+8t \\ 3t+1-3t-1-4t \\ -9t-3+3t+1+12t \end{pmatrix} = \begin{pmatrix} 4t \\ -2t \\ 3t-1 \end{pmatrix}$$
- M1 A2
- $$\therefore x = 4t, y = -2t, z = 3t - 1$$
- giving $(t =) \frac{x}{4} = \frac{y}{-2} = \frac{z+1}{3}$ M1 A1 (13)
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Total (75)

Performance Record – FP3 Paper E