GCE Examinations

Further Pure Mathematics Module FP3

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. Given that

$$y = \frac{1}{1-x},$$

prove by induction that $\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n} = \dfrac{n!}{(1-x)^{n+1}}$ for all integers n, $n \geq 1$. **(7 marks)**

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2. The variable y satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y + 2, \qquad y = 0 \text{ at } x = 0.$$

(a) Given that $y \approx 2h$ when $x = h$, use the approximation $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \dfrac{y_1 - y_{-1}}{2h}$ once to obtain an estimate for y as a function of h when $x = 2h$.

(4 marks)

(b) Use the same approximation to show that an estimate for y when $x = 3h$ is given by

$$y \approx 2h(2h^3 + 8h^2 + 4h + 3).$$ **(3 marks)**

(c) Hence find an estimate for y when $x = 0.3$ **(2 marks)**

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3. Given that

$$z^6 - z^3\sqrt{3} + 1 = 0,$$

(a) find the possible values of z^3, giving your answers in the form $x + \mathrm{i}y$ where $x, y \in \mathbb{R}$.

(3 marks)

(b) Hence find all possible values of z in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$.

(7 marks)

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4. *(a)* Write down the first three terms of the series of e^{x^2}, in ascending powers of x.

(2 marks)

(b) Hence, or otherwise, find the series expansion, in ascending powers of x up to and including the term in x^4, of

$$\frac{e^{x^2}}{1+2x}.$$

(5 marks)

(c) Hence find an estimate for the area of the region bounded by the x-axis, the lines $x = 0$ and $x = 0.2$, and the curve

$$y = \frac{e^{x^2}}{1+2x},$$

giving your answer to 3 significant figures. **(4 marks)**

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5. The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 2 & a & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{pmatrix}.$$

(a) Find $\mathbf{A}^{-1}$, showing your working clearly and stating the condition for which $\mathbf{A}$ is non-singular.

(7 marks)

Relative to a fixed origin O, the transformation T maps the point P onto the point Q. When $a = -1$, Q has position vector $5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

(b) Find the position vector of P, showing your working clearly. **(4 marks)**

Turn over

6. The planes Π_1 and Π_2 are defined by the equations $2x - y + 3z = 5$ and $x + 4y + z = -2$ respectively.

(a) Find, to the nearest degree, the acute angle between Π_1 and Π_2. **(4 marks)**

The point A has coordinates $(2, 1, -2)$.

(b) Find the perpendicular distance between A and Π_1. **(4 marks)**

The plane Π_3 is perpendicular to Π_1 and Π_2 and the point with coordinates $(0, 4, -1)$ lies on Π_3.

(c) Find the equation of Π_3 in the form $ax + by + cz = d$. **(5 marks)**

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7. The transformation T from the complex z-plane to the complex w-plane is given by

$$w = \frac{1}{z^* - 2}, \quad z \neq 2.$$

(a) Show that the image in the w-plane of the line $\text{Re}\,(z) = 5$ in the z-plane, under T, is a circle. Find its centre and radius.

(7 marks)

The region represented by $\text{Re}\,(z) > 5$ in the z-plane is transformed under T into the region represented by R in the w-plane.

(b) Show the region R on an Argand diagram. **(3 marks)**

(c) Find the image in the w-plane under T of the half-line $\arg(z - 2) = \frac{\pi}{4}$ in the the z-plane.

(4 marks)

END