

GCE Examinations
Advanced Subsidiary / Advanced Level
Further Pure Mathematics
Module FP3

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



Written by Rosemary Smith & Shaun Armstrong

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FP3 Paper C – Marking Guide

1. (a) $\frac{y_1 - y_0}{0.01} = e^{x_0} \cosh(2y_0 + x_0) \therefore y_1 = 0.01 e^{x_0} \cosh(2y_0 + x_0) + y_0$ M1 A1
 $x_0 = 1, x_1 = 1.01; y_0 = 1 \therefore y_1 = 1.2736\dots = 1.27$ (3sf) A1
- (b) $\frac{y_1 - y_{-1}}{0.02} = e^{x_0} \cosh(2y_0 + x_0) \therefore y_{-1} = y_1 - 0.02 e^{x_0} \cosh(2y_0 + x_0)$ M1 A1
 $x_{-1} = 0.99, x_0 = 1, x_1 = 1.01; y_0 = 1, y_1 = 1.2736\dots, y_{-1} = ?$
 $\therefore y_{-1} = 0.7263\dots = 0.726$ (3sf) A1 (6)
-
2. (a) $\vec{AB} = -4\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \vec{AC} = (a-2)\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$ B1
 $\vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & -1 \\ a-2 & -6 & 2 \end{vmatrix}$
 $= \mathbf{i}(6-6) - \mathbf{j}(-8+a-2) + \mathbf{k}[24-3(a-2)] = (10-a)\mathbf{j} + 3(10-a)\mathbf{k}$ M1 A2
- (b) area = $\frac{1}{2} |\vec{AB} \times \vec{AC}| = 4\sqrt{10}$ M1
 $\therefore |10-a| \times \sqrt{1+9} = 8\sqrt{10}$ M1
 $|10-a| = 8$ so $a = 2$ or 18 A1 (7)
-
3. (a) $z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos(-n\theta) + i \sin(-n\theta)$ M1
 $= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta = 2 \cos n\theta$ A1
- (b) dividing by z^2 gives $5z^2 - 11z + 16 - \frac{11}{z} + \frac{5}{z^2} = 0$ M1
 $\therefore 5(2\cos 2\theta) - 11(2\cos \theta) + 16 = 0$ M1
 $5\cos 2\theta - 11\cos \theta + 8 = 0$ A1
 $5(2\cos^2 \theta - 1) - 11\cos \theta + 8 = 0$ M1
 $10\cos^2 \theta - 11\cos \theta + 3 = 0$
 $(5\cos \theta - 3)(2\cos \theta - 1) = 0$ M1
 $\therefore \cos \theta = \frac{3}{5}$ or $\frac{1}{2}$ A1
 if $\cos \theta = \frac{3}{5}, \sin \theta = \pm \frac{4}{5}$; if $\cos \theta = \frac{1}{2}, \sin \theta = \pm \frac{\sqrt{3}}{2}$ M1
 $\therefore z = \frac{3}{5} \pm \frac{4}{5}i$ or $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ A1 (10)

4. (a) assume true for $n = k \therefore \mathbf{A}^k = \begin{pmatrix} 1 & k & \frac{1}{2}k(k+1) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}$

$\therefore \mathbf{A}^{k+1} = \begin{pmatrix} 1 & k & \frac{1}{2}k(k+1) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1+k & 1+k+\frac{1}{2}k(k+1) \\ 0 & 1 & 1+k \\ 0 & 0 & 1 \end{pmatrix}$ M1 A1

$1 + k + \frac{1}{2}k(k+1) = \frac{1}{2}(k+1)(2+k) = \frac{1}{2}(k+1)[(k+1)+1]$ M1

$\therefore \mathbf{A}^{k+1} = \begin{pmatrix} 1 & k+1 & \frac{1}{2}(k+1)[(k+1)+1] \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$ A1

\therefore true for $n = k + 1$ if true for $n = k$

if $n = 1 \mathbf{A}^1 = \begin{pmatrix} 1 & 1 & \frac{1}{2} \times 1 \times 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \therefore$ true for $n = 1$ B1

\therefore by induction true for $n \in \mathbb{Z}^+$ A1

(b) $\det \mathbf{A}^n = 1(1-0) - n(0-0) + \frac{1}{2}n(n+1)(0-0) = 1$ M1 A1

matrix of cofactors: $\begin{pmatrix} 1 & 0 & 0 \\ -n & 1 & 0 \\ \frac{1}{2}n(n-1) & -n & 1 \end{pmatrix} [n^2 - \frac{1}{2}n(n+1) = \frac{1}{2}n(n-1)]$ M1 A1

$\therefore (\mathbf{A}^n)^{-1} = \begin{pmatrix} 1 & -n & \frac{1}{2}n(n-1) \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$ A1 (11)

5. (a) let $y = \arccos x \therefore \cos y = x$

$-\sin y \frac{dy}{dx} = 1$ M1

$\frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}} \therefore f'(x) = \frac{-1}{(1-x^2)^{\frac{1}{2}}}$ M1 A1

(b) $f''(x) = \frac{1}{2}(-2x)(1-x^2)^{-\frac{3}{2}} = \frac{-x}{(1-x^2)^{\frac{3}{2}}}$ M1 A1

$\therefore (1-x^2)f''(x) - xf'(x) = (1-x^2) \times \frac{-x}{(1-x^2)^{\frac{3}{2}}} + \frac{x}{(1-x^2)^{\frac{1}{2}}}$

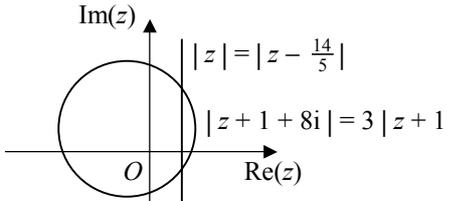
$= \frac{-x}{(1-x^2)^{\frac{1}{2}}} + \frac{x}{(1-x^2)^{\frac{1}{2}}} = 0$ A1

(c) $(1-x^2)f'''(x) - 2xf''(x) - xf'(x) = 0$ M1 A1

$f(0) = \frac{\pi}{2}, f'(0) = -1, f''(0) = 0, f'''(0) = -1$ A1

$\therefore f(x) = \frac{\pi}{2} - 1x + 0 - 1(\frac{1}{3!})x^3 + \dots = \frac{\pi}{2} - x - \frac{1}{6}x^3 + \dots$ M1 A1 (11)

6. (a)
$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & -\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$
 M1
- $(2-\lambda)[- \lambda(2-\lambda) - 1] + 1[-(2-\lambda) - 1] + 1(-1 + \lambda) = 0$ A1
- $(2-\lambda)(\lambda^2 - 2\lambda - 1) - (2-\lambda) - 1 + \lambda = 0$ M1
- $(2-\lambda)(\lambda^2 - 2\lambda - 1) - (2-\lambda) - (2-\lambda) = 0$
- $(2-\lambda)(\lambda^2 - 2\lambda - 1 - 2) = (2-\lambda)(\lambda^2 - 2\lambda - 3) = 0$ A1
- $(2-\lambda)(\lambda - 3)(\lambda + 1) = 0$ M1
- $\therefore \lambda = 2$ is an eigenvalue, also $\lambda = -1$ or 3 A2
- (b) $\lambda = 2, \begin{pmatrix} 0 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ M1
- $-y + z = 0 \therefore y = z; \quad x + y = 0 \therefore x = -y$ M1 A1
- \therefore eigenvector $k \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ A1
- (c) $\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ M1 A2 (14)

7. (a) $|x + iy + 1 + 8i| = 3|x + iy + 1|$ M1 A1
- $(x+1)^2 + (y+8)^2 = 9(x+1)^2 + 9y^2$ M1
- $y^2 + 16y + 64 = 8(x+1)^2 + 9y^2$
- $8(x+1)^2 + 8y^2 - 16y - 64 = 0$
- $(x+1)^2 + y^2 - 2y - 8 = 0$
- $(x+1)^2 + (y-1)^2 - 1 - 8 = 0$ M1
- $(x+1)^2 + (y-1)^2 = 9$ A1
- \therefore circle, centre $-1 + i$, radius 3 A2
- (b) $|z| = |z - \frac{14}{5}|$ is perp. bisector of 0 and $\frac{14}{5}$ i.e. $\text{Re}(z) = \frac{7}{5}$ B1
-  B1
- $|z + 1 + 8i| = 3|z + 1|$ B2
- (c) intersect when $x = \frac{7}{5}$ M1
- $\therefore (\frac{7}{5} + 1)^2 + (y - 1)^2 = 9$ A1
- $(y - 1)^2 = 9 - \frac{144}{25} = \frac{81}{25}$ M1
- $\therefore y = 1 \pm \frac{9}{5}$ A1
- intersect at $\frac{7}{5} - \frac{4}{5}i$ and $\frac{7}{5} + \frac{14}{5}i$ A1 (16)

Total (75)

