

GCE Examinations
Advanced Subsidiary / Advanced Level
Further Pure Mathematics
Module FP2

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP2 Paper H – Marking Guide

1. $2x^2 + y^2 = 4, \quad 4x + 2y \frac{dy}{dx} = 0 \quad \text{or} \quad 2x + y \frac{dy}{dx} = 0$ M1 A1

$$2 + y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \text{M1 A1}$$

at $(1, -\sqrt{2}) \quad 2 - \sqrt{2} \frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = \sqrt{2}$ A1

$$2 - \sqrt{2} \frac{d^2y}{dx^2} + 2 = 0 \quad \therefore \frac{d^2y}{dx^2} = 2\sqrt{2} \quad \text{A1}$$

$$\rho = \frac{(1+(\sqrt{2})^2)^{\frac{3}{2}}}{2\sqrt{2}} = \frac{3\sqrt{3}}{2\sqrt{2}} = \frac{3\sqrt{6}}{4} \quad \text{M1 A1} \quad \textcolor{red}{(8)}$$

2. (a) $f(x) = \cosh x = \frac{1}{2}(e^x + e^{-x})$

$$f(-x) = \frac{1}{2}(e^{-x} + e^{-(x)}) = \frac{1}{2}(e^{-x} + e^x) = f(x) \quad \therefore \cosh x \text{ is even} \quad \text{M1 A1}$$

(b) $\ln(xy) = \operatorname{arccosh}(\frac{5}{3}) = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9}-1}\right) = \ln 3 \quad \text{M1 A1}$

$$\therefore xy = 3 \quad \text{A1}$$

$$\cosh(3x-y) = 1 \quad \therefore 3x-y = 0 \quad \text{B1}$$

$$\therefore 3x = y = \frac{3}{x}, \quad x^2 = 1 \quad \text{M1}$$

$$x > 0 \quad \therefore x = 1, y = 3 \quad \text{A1} \quad \textcolor{red}{(8)}$$

3. $\int \frac{1}{13 \cosh x - 5 \sinh x} dx = \int \frac{1}{\frac{13}{2}(e^x + e^{-x}) - \frac{5}{2}(e^x - e^{-x})} dx \quad \text{M1}$

$$= \int \frac{1}{4e^x + 9e^{-x}} dx \quad \text{A1}$$

$$= \int \frac{e^x}{4e^{2x} + 9} dx \quad \text{M1}$$

$$u = e^x, \quad \frac{du}{dx} = e^x \quad \text{M1}$$

$$= \int \frac{1}{4u^2 + 9} du \quad \text{A1}$$

$$= \frac{1}{4} \int \frac{1}{u^2 + \frac{9}{4}} du \quad \text{M1}$$

$$= \frac{1}{4} \times \frac{2}{3} \arctan\left(\frac{2u}{3}\right) + c \quad \text{A1}$$

$$= \frac{1}{6} \arctan\left(\frac{2}{3}e^x\right) + c \quad \text{A1} \quad \textcolor{red}{(8)}$$

4. (a) let $y = \arcsin(2x - 1) \therefore \sin y = 2x - 1$
- $$\therefore \cos y \frac{dy}{dx} = 2 \quad \text{M1}$$
- $$\begin{aligned} \frac{dy}{dx} &= \frac{2}{\sqrt{1-\sin^2 y}} = \frac{2}{\sqrt{1-(2x-1)^2}} && \text{M1 A1} \\ &= \frac{2}{\sqrt{1-(4x^2-4x+1)}} = \frac{2}{\sqrt{4x-4x^2}} = \frac{1}{\sqrt{x-x^2}} && \text{A1} \end{aligned}$$
- (b) $x = \frac{3}{4}, y = \arcsin \frac{1}{2} = \frac{\pi}{6} \quad \text{B1}$
- $$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{3}{4}-\frac{9}{16}}} = \frac{1}{\sqrt{\frac{3}{16}}} = \frac{4}{\sqrt{3}} \quad \text{M1}$$
- eqn. is $y - \frac{\pi}{6} = \frac{4}{\sqrt{3}}(x - \frac{3}{4}) \quad \text{M1}$
- $$x = 0 \therefore y - \frac{\pi}{6} = \frac{4}{\sqrt{3}} \times (-\frac{3}{4}) \quad \text{M1}$$
- $$y = \frac{\pi}{6} - \sqrt{3} \quad \text{A1} \quad \text{(9)}$$
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5. (a) $2y \frac{dy}{dx} = 4a \therefore \frac{dy}{dx} = \frac{2a}{y} \quad \text{M1}$
- at P , $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t} \quad \text{A1}$
- eqn. is $y - 2at = \frac{1}{t}(x - at^2) \quad \text{M1}$
giving $yt = x + at^2 \quad \text{A1}$
- (b) at Q , $y = 0 \therefore x = -at^2$; at R , $x = 0 \therefore y = at \quad \text{M1 A1}$
 $\therefore M$ is $(-\frac{1}{2}at^2, \frac{1}{2}at) \quad \text{A1}$
- (c) grad of $OM = \frac{\frac{1}{2}at-0}{-\frac{1}{2}at^2-0} = -\frac{1}{t}; \text{ grad of } OP = \frac{2at-0}{at^2-0} = \frac{2}{t} \quad \text{M1 A1}$
 $OM \perp OP \therefore \frac{2}{t} \times -\frac{1}{t} = -1 \therefore t^2 = 2 \quad \text{M1 A1} \quad \text{(11)}$
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6. (a) $I_n - I_{n-2} = \int \frac{\cos n\theta - \cos(n-2)\theta}{\sin \theta} d\theta$ M1
 $= \int \frac{-2 \sin(n-1)\theta \times \sin \theta}{\sin \theta} d\theta$ M1 A1
 $= \int -2 \sin(n-1)\theta d\theta$
 $= \frac{2}{n-1} \cos(n-1)\theta$ M1
 $\therefore I_n = \frac{2}{n-1} \cos(n-1)\theta + I_{n-2}$ A1

(b) $I_n = \left[\frac{2}{n-1} \cos(n-1)\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_{n-2}$
 $I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos \theta}{\sin \theta} d\theta = [\ln |\sin \theta|]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ M1 A1
 $= \ln 1 - \ln \frac{1}{\sqrt{2}} = -\ln 2^{-\frac{1}{2}} = \frac{1}{2} \ln 2$ M1 A1
 $I_3 = \left[\frac{2}{2} \cos 2\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_1 = -1 - 0 + \frac{1}{2} \ln 2 = \frac{1}{2} \ln 2 - 1$ M1 A1
 $I_5 = \left[\frac{2}{4} \cos 4\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_3 = \frac{1}{2} - \left(-\frac{1}{2} \right) + \frac{1}{2} \ln 2 - 1 = \frac{1}{2} \ln 2$ M1 A1 **(13)**

7. (a) $ae = \sqrt{3}$, $\frac{a}{e} = \frac{4}{\sqrt{3}}$ $\therefore ae \times \frac{a}{e} = a^2 = 4$ M1
 $a > 0 \therefore a = 2$ A1
 $b^2 = a^2(1 - e^2) = a^2 - (ae)^2 = 4 - 3 = 1$ M1
 $b > 0 \therefore b = 1$ A1

(b) $\frac{x^2}{4} + y^2 = 1$, $\frac{1}{2}x + 2y \frac{dy}{dx} = 0 \therefore \frac{dy}{dx} = \frac{-x}{4y}$ M1 A1
 $A = \int_{-2}^2 2\pi y \sqrt{1 + \frac{x^2}{16y^2}} dx$ M1 A1
 $= 2\pi \int_{-2}^2 y \sqrt{\frac{16y^2 + x^2}{16y^2}} dx$
 $= 2\pi \int_{-2}^2 \frac{y}{4y} \sqrt{16(1 - \frac{x^2}{4}) + x^2} dx$ M1
 $= \frac{\pi}{2} \int_{-2}^2 \sqrt{16 - 4x^2 + x^2} dx$
 $= \frac{\pi}{2} \int_{-2}^2 \sqrt{16 - 3x^2} dx$ A1

(c) $3x^2 = 16 \sin^2 \theta$, $x = \frac{4}{\sqrt{3}} \sin \theta$, $\frac{dx}{d\theta} = \frac{4}{\sqrt{3}} \cos \theta$ M1 A1
 $A = \frac{\pi}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{16 - 16 \sin^2 \theta} \times \frac{4}{\sqrt{3}} \cos \theta d\theta$ M1
 $= \frac{\pi}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4 \cos \theta \times \frac{4}{\sqrt{3}} \cos \theta d\theta$
 $= \frac{8\pi}{\sqrt{3}} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2 \theta d\theta$ A1
 $= \frac{4\pi}{\sqrt{3}} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 + \cos 2\theta d\theta$ M1
 $= \frac{4\pi}{\sqrt{3}} [\theta + \frac{1}{2} \sin 2\theta]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$ A1
 $= \frac{4\pi}{\sqrt{3}} [\frac{\pi}{3} + \frac{\sqrt{3}}{4} - (-\frac{\pi}{3} - \frac{\sqrt{3}}{4})] = \frac{4\pi}{\sqrt{3}} (\frac{2\pi}{3} + \frac{\sqrt{3}}{2}) = \frac{8}{9} \pi^2 \sqrt{3} + 2\pi$ M1 A1 **(18)**

Total **(75)**

Performance Record – FP2 Paper H