GCE Examinations

Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



Written by Rosemary Smith & Shaun Armstrong © Solomon Press

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1. A student without a calculator must find the value of x given that artanh $x = \ln 3$.

With clear working, show how the student could find x and state the value he should obtain.

(4 marks)

$$f(x) = \sin 2x - x \cosh^2 x.$$

- (a) Find f'(x).
- (b) Show that the curve with equation y = f(x) has a stationary point in the interval $0.3 \le x \le 0.4$.

(3 marks)

(3 marks)

3. Given that

$$\int_{0}^{\frac{2\pi}{3}} \frac{1}{5+4\cos x} \, \mathrm{d}x = a\pi, \qquad a \in \mathbb{Q},$$

use the substitution $t = tan(\frac{1}{2}x)$ to find the value of *a*. (9 marks)

4. The curve C has equation $y = a \cosh\left(\frac{x}{a}\right)$, where a is a positive constant.

The area bounded by the curve *C*, the *x*-axis and the lines x = -a and x = a is rotated through 2π radians about the *x*-axis.

Show that the curved surface area of the solid generated is $\pi a^2(\sinh 2 + 2)$. (9 marks)

5. The intrinsic equation of the curve C is $s = 2\psi$.

Given that *s* is measured from the origin,

- (a) find a Cartesian equation of C, (9 marks)
- (b) sketch C. (2 marks)

6. (a) Using the definitions of hyperbolic functions in terms of exponential functions, prove that

$$\cosh(x+y) \equiv \cosh x \cosh y + \sinh x \sinh y.$$
 (4 marks)

Given that

$$5\cosh x + 4\sinh x \equiv R\cosh(x + \alpha),$$

find

- (b) the value of R, (3 marks)
- (c) the value of α , giving your answer in terms of natural logarithms. (3 marks)
- (d) Hence, or otherwise, state the minimum value of $5 \cosh x + 4 \sinh x$. (1 mark)

7.
$$I_n = \int_0^1 x^n e^{x^2} dx, \quad n \ge 0.$$

(a) Show that

$$I_n = \frac{1}{2}e - \frac{1}{2}(n-1)I_{n-2}, \quad n \ge 2.$$
 (5 marks)

(b) Hence find

$$I_n = \int_0^1 x^5 \mathrm{e}^{x^2} \, \mathrm{d}x,$$

giving your answer in terms of e.

- 8. The line with equation y = mx + c is a tangent to the parabola with equation $y^2 = 8x$.
 - (a) Show that mc = 2. (5 marks)

The lines l_1 and l_2 are tangents to both the parabola with equation $y^2 = 8x$ and the circle with equation $x^2 + y^2 = 2$.

(b) Find the equations of l_1 and l_2 .

END

(6 marks)

(9 marks)