GCE Examinations

Further Pure Mathematics Module FP2

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1.
$$y = \frac{\operatorname{cosech} x}{x^2 + 1}.$$

(a) Find
$$\frac{dy}{dx}$$
. (4 marks)

(b) Find the value of $\frac{dy}{dx}$ when x = 0.5, giving your answer to 2 decimal places. (1 mark)

2. A curve has intrinsic coordinates (s, ψ) and radius of curvature ρ .

Given that $\rho = 2(s + a)$, where *a* is constant, show that the intrinsic equation of the curve can be written in the form

$$s = A e^{2\psi} - a ,$$

where A is constant.

3. (a) Prove that

$$\sinh 3x \equiv 4 \sinh^3 x + 3 \sinh x \,. \tag{5 marks}$$

(5 marks)

(b) Hence, or otherwise, solve the equation

 $\sinh 3x = 7 \sinh^2 x \, ,$

giving your answers in terms of natural logarithms where appropriate. (6 marks)

4. (a) Find $\int \frac{1}{\sqrt{9-4x^2}} \, dx$. (3 marks)

(b) Find
$$\int \frac{1-2x}{\sqrt{9-4x^2}} \, dx$$
. (3 marks)

(c) Hence, or otherwise, solve the differential equation

$$\sqrt{9-4x^2}\,\frac{\mathrm{d}y}{\mathrm{d}x} = y(1-2x),$$

given that
$$y = 1$$
 when $x = 0$. (6 marks)

- 5. The curve C has equation $y^2 = 4ax$, where a is a positive constant.
 - (a) Show that an equation of the tangent to C at the point $P(ap^2, 2ap), p \neq 0$, is

$$yp = x + ap^2.$$
 (4 marks)

The point $Q(aq^2, 2aq)$, is on C where $q \neq 0$ and $p \neq q$. The chord PQ passes through the focus of C.

Show that

- (b) pq = -1, (5 marks)
- (c) the tangent to C at P and the tangent to C at Q meet on the directrix of C. (4 marks)

6.
$$I_n = \int_0^{\frac{\pi}{4}} \sec^n x \, dx, \quad n \ge 0.$$

(a) Show that

$$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}, \quad n \ge 2.$$
 (7 marks)

(b) Hence find the exact value of I_3 , giving your answer in terms of natural logarithms.

(6 marks)

7. (*a*) Show that

$$\int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \operatorname{arsinh}\left(\frac{x}{a}\right) + c.$$
 (9 marks)

The parametric equations of the curve C are

$$x = 2t, \qquad y = t^2, \qquad 0 \le t \le 3.$$

(b) Show that the length of C is given by

$$2\int_{0}^{3} \sqrt{1+t^{2}} dt$$
. (4 marks)

(c) Find the length of C.

END

(3 marks)