

**GCE Examinations**  
**Advanced Subsidiary / Advanced Level**  
**Further Pure Mathematics**  
**Module FP2**

**Paper B**

**MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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## FP2 Paper B – Marking Guide

1.  $\frac{-y}{\sqrt{1-x^2}} + \arccos x \frac{dy}{dx} - \frac{1}{\pi} e^{2x} - \frac{2x}{\pi} e^{2x} = 0$  M1 A2

when  $x = 0$ ,  $y \times \frac{\pi}{2} - 0 - 1 = 0 \therefore y = \frac{2}{\pi}$  B1

when  $x = 0$ ,  $-\frac{2}{\pi} + \frac{\pi}{2} \frac{dy}{dx} - \frac{1}{\pi} - 0 = 0$  M1 A1

$\therefore \frac{\pi}{2} \frac{dy}{dx} = \frac{3}{\pi}$  so  $\frac{dy}{dx} = \frac{6}{\pi^2}$  A1 (7)

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2.  $f'(x) = 5 \sinh x + 3 \cosh x$  M1

S.P.  $\therefore 5 \sinh x + 3 \cosh x = 0$  giving  $\tanh x = -\frac{3}{5}$  M1 A1

$x = \operatorname{artanh}(-\frac{3}{5}) = \frac{1}{2} \ln \left( \frac{1-\frac{3}{5}}{1+\frac{3}{5}} \right)$  M1 A1

$x = \frac{1}{2} \ln \frac{1}{4} = -\ln 2$  A1

$f(-\ln 2) = 5 \cosh(-\ln 2) + 3 \sinh(-\ln 2) = 4$  M1

$\therefore p = -1, q = 2, r = 4$  A1 (8)

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3. (a)  $x(mx + c) = -9 \therefore mx^2 + cx + 9 = 0$  M1 A1

tangent  $\therefore b^2 - 4ac = 0 \therefore c^2 - 4 \times m \times 9 = 0$  M1

$\therefore c^2 = 36m$  giving  $c = \pm 6\sqrt{m}$  A1

(b) (4, -2)  $\therefore -2 = 4m + c$  and  $c^2 = 36m$  M1

$\therefore (-2 - 4m)^2 = 36m$

$4 + 16m + 16m^2 = 36m$

giving  $4m^2 - 5m + 1 = 0$

$(4m - 1)(m - 1) = 0$

$m = \frac{1}{4}$  or 1

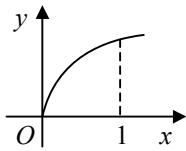
if  $m = \frac{1}{4}, c = -3$ ; if  $m = 1, c = -6$  M1

$\therefore$  tangents are  $y = \frac{1}{4}x - 3$  and  $y = x - 6$  A1 (9)

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4.  $y^2 = x \therefore 2y \frac{dy}{dx} = 1$  so  $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

M1 A1



$\therefore$  lower limit = 0

M1

$$A = \int_0^1 2\pi y \sqrt{1+\frac{1}{4x}} dx \quad \text{M1 A1}$$

$$= \int_0^1 2\pi \sqrt{x} \sqrt{1+\frac{1}{4x}} dx = \int_0^1 \pi \sqrt{4x+1} dx \quad \text{M1 A1}$$

$$= \pi \left[ \frac{2}{3} \times \frac{1}{4} (4x+1)^{\frac{3}{2}} \right]_0^1 \quad \text{M1 A1}$$

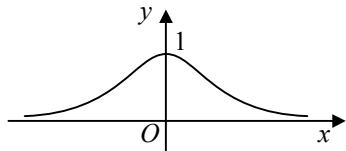
$$= \frac{1}{6} \pi [5^{\frac{3}{2}} - 1^{\frac{3}{2}}] = \frac{1}{6} \pi (5\sqrt{5} - 1) \quad \text{M1 A1} \quad \text{(11)}$$


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5. (a)  $\cosh x = \frac{1}{2}(e^x + e^{-x}) \therefore \operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

B1

(b)



B2

$$(c) \int \operatorname{sech} x dx = \int \frac{2}{e^x + e^{-x}} dx = \int \frac{2e^x}{e^{2x} + 1} dx$$

$$u = e^x \therefore \frac{du}{dx} = e^x \quad \text{M1}$$

$$I = \int \frac{2}{u^2 + 1} du \quad \text{A1}$$

$$= 2 \arctan u + c = 2 \arctan e^x + c \quad \text{M1 A1}$$

$$(d) V = \int_{-a}^a \pi \operatorname{sech}^2 x dx \quad \text{M1}$$

$$= [\pi \tanh x]_{-a}^a \quad \text{A1}$$

$$= \pi[\tanh a - \tanh(-a)] = 2\pi \tanh a \quad \text{M1 A1}$$

(e) as  $a \rightarrow \infty$ ,  $\tanh a \rightarrow 1$ ,  $V \rightarrow 2\pi$   $\therefore$  limit of volume is  $2\pi$  A1 **(12)**

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6. (a)  $u = (2 - x^2)^n, u' = -2nx(2 - x^2)^{n-1}; v' = 1, v = x$  M1  
 $I_n = [x(2 - x^2)^n]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} -2nx^2(2 - x^2)^{n-1} dx$  A1

$$I_n = [0 - 0] - 2n \int_0^{\sqrt{2}} (-x^2 + 2 - 2)(2 - x^2)^{n-1} dx \quad \text{M2 A1}$$

$$I_n = -2n \int_0^{\sqrt{2}} (2 - x^2)(2 - x^2)^{n-1} dx - 2n \int_0^{\sqrt{2}} -2(2 - x^2)^{n-1} dx \quad \text{M1}$$

$$I_n = -2n \int_0^{\sqrt{2}} (2 - x^2)^n dx + 4n \int_0^{\sqrt{2}} (2 - x^2)^{n-1} dx \quad \text{A1}$$

$$I_n = -2nI_n + 4nI_{n-1} \quad \text{M1}$$

$$(1 + 2n)I_n = 4nI_{n-1}$$

$$I_n = \frac{4n}{2n+1} I_{n-1} \quad \text{A1}$$

(b)  $I_0 = \int_0^{\sqrt{2}} dx = [x]_0^{\sqrt{2}} = \sqrt{2}$  B1

$$I_1 = \frac{4}{3} I_0 = \frac{4}{3} \sqrt{2} \quad \text{M1}$$

$$I_2 = \frac{8}{5} I_1 = \frac{8}{5} \times \frac{4}{3} \sqrt{2} \quad \text{M1}$$

$$I_3 = \frac{12}{7} I_2 = \frac{12}{7} \times \frac{8}{5} \times \frac{4}{3} \sqrt{2} = \frac{128}{35} \sqrt{2} \quad \text{A1} \quad \text{(13)}$$


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7. (a)  $\rho = \frac{ds}{d\psi} = \frac{\frac{1}{2} \sec^2 \frac{1}{2}\psi}{\tan \frac{1}{2}\psi}$  M1 A1  
 $= \frac{1}{2} \times \frac{1}{\cos^2 \frac{1}{2}\psi} \times \frac{\cos \frac{1}{2}\psi}{\sin \frac{1}{2}\psi} = \frac{1}{2 \cos \frac{1}{2}\psi \sin \frac{1}{2}\psi} = \frac{1}{\sin \psi} = \operatorname{cosec} \psi$  M1 A1

(b)  $\frac{ds}{d\psi} = \operatorname{cosec} \psi, \frac{dy}{ds} = \sin \psi$

$$\frac{dy}{d\psi} = \frac{dy}{ds} \frac{ds}{d\psi} = 1 \quad \therefore y = \psi + c \quad \text{M1 A1}$$

$$y = \frac{\pi}{2}, \psi = \frac{\pi}{2} \quad \therefore c = 0 \text{ so } y = \psi \quad \text{M1 A1}$$

(c)  $\frac{dy}{dx} = \tan \psi = \tan y$  M1

$$\int_{\frac{\pi}{2}}^y \cot y dy = \int_0^x dx \quad \text{M1 A1}$$

$$[\ln |\sin y|]_{\frac{\pi}{2}}^y = [x]_0^x \quad \text{M1 A1}$$

$$\ln(\sin y) - \ln 1 = x - 0 \quad [0 < y \leq \frac{\pi}{2} \quad \therefore \sin y > 0] \quad \text{M1}$$

$$x = \ln(\sin y) \quad \text{A1} \quad \text{(15)}$$


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Total (75)

## **Performance Record – FP2 Paper B**