## GCE Examinations

## Advanced / Advanced Subsidiary

## Core Mathematics C2

## Paper L

## Time: 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72 .
- You are reminded of the need for clear presentation in your answers.

1. (i) Sketch on the same diagram the graphs of $y=\sin 2 x$ and $y=\tan \frac{x}{2}$ for $x$ in the interval $0 \leq x \leq 360^{\circ}$.
(ii) Hence state how many solutions exist to the equation

$$
\sin 2 x=\tan \frac{x}{2}
$$

for $x$ in the interval $0 \leq x \leq 360^{\circ}$ and give a reason for your answer.
2.


The diagram shows a circle of radius $r$ and centre $O$ in which $A D$ is a diameter.
The points $B$ and $C$ lie on the circle such that $O B$ and $O C$ are arcs of circles of radius $r$ with centres $A$ and $D$ respectively.

Show that the area of the shaded region $O B C$ is $\frac{1}{6} r^{2}(3 \sqrt{3}-\pi)$.
3. The sequence $u_{1}, u_{2}, u_{3}, \ldots$ is defined by

$$
u_{n+1}=\left(u_{n}\right)^{2}-1, \quad n \geq 1 .
$$

Given that $u_{1}=k$, where $k$ is a constant,
(i) find expressions for $u_{2}$ and $u_{3}$ in terms of $k$.

Given also that $u_{2}+u_{3}=11$,
(ii) find the possible values of $k$.
4.


The diagram shows the curve with equation $y=\frac{1}{x^{2}+1}$.
The shaded region $R$ is bounded by the curve, the coordinate axes and the line $x=2$.
(i) Use the trapezium rule with four strips of equal width to estimate the area of $R$.

The cross-section of a support for a bookshelf is modelled by $R$ with 1 unit on each axis representing 8 cm . Given that the support is 2 cm thick,
(ii) find an estimate for the volume of the support.
5. (i) Find the value of $a$ such that

$$
\begin{equation*}
\log _{a} 27=3+\log _{a} 8 . \tag{3}
\end{equation*}
$$

(ii) Solve the equation

$$
\begin{equation*}
2^{x+3}=6^{x-1} \tag{4}
\end{equation*}
$$

giving your answer to 3 significant figures.
6. (i) Evaluate

$$
\begin{equation*}
\int_{2}^{4}\left(2-\frac{1}{x^{2}}\right) \mathrm{d} x . \tag{4}
\end{equation*}
$$

(ii) Given that

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x^{3}+1 \tag{5}
\end{equation*}
$$

and that $y=3$ when $x=0$, find the value of $y$ when $x=2$.
7.


The diagram shows part of the curve $y=\mathrm{f}(x)$ where $\mathrm{f}(x)=\frac{1-8 x^{3}}{x^{2}}, x \neq 0$.
(i) Solve the equation $\mathrm{f}(x)=0$.
(ii) Find $\int \mathrm{f}(x) \mathrm{d} x$.
(iii) Find the area of the shaded region bounded by the curve $y=\mathrm{f}(x)$, the $x$-axis and the line $x=2$.
8. A store begins to stock a new range of DVD players and achieves sales of $£ 1500$ of these products during the first month. In a model it is assumed that sales will decrease by $£ x$ in each subsequent month, forming an arithmetic sequence.

Given that sales total $£ 8100$ during the first six months, use the model to
(i) find the value of $x$,
(ii) find the expected value of sales in the eighth month,
(iii) show that the expected total of sales in pounds during the first $n$ months is given by $k n(51-n)$, where $k$ is an integer to be found.
(iv) Explain why this model cannot be valid over a long period of time.
9.

$$
\mathrm{f}(x)=2 x^{3}-5 x^{2}+x+2
$$

(i) Show that $(x-2)$ is a factor of $\mathrm{f}(x)$.
(ii) Fully factorise $\mathrm{f}(x)$.
(iii) Solve the equation $\mathrm{f}(x)=0$.
(iv) Find, in terms of $\pi$, the values of $\theta$ in the interval $0 \leq \theta \leq 2 \pi$ for which

$$
\begin{equation*}
2 \sin ^{3} \theta-5 \sin ^{2} \theta+\sin \theta+2=0 \tag{4}
\end{equation*}
$$

