## GCE Examinations

## Advanced Subsidiary

## Core Mathematics C1

## Paper E

Time: 1 hour 30 minutes

## Instructions and Information

Candidates may NOT use a calculator in this paper
Full marks may be obtained for answers to ALL questions.
Mathematical formulae and statistical tables are available.
This paper has ten questions.

## Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

1. (a) Express $\frac{18}{\sqrt{3}}$ in the form $k \sqrt{3}$.
(2)
(b) Express $(1-\sqrt{3})(4-2 \sqrt{3})$ in the form $a+b \sqrt{3}$ where $a$ and $b$ are integers.
(2)
2. Solve the equation

$$
\begin{equation*}
3 x-\frac{5}{x}=2 . \tag{4}
\end{equation*}
$$

3. The straight line $l$ has the equation $x-5 y=7$.

The straight line $m$ is perpendicular to $l$ and passes through the point $(-4,1)$.
Find an equation for $m$ in the form $y=m x+c$.
4. A sequence of terms is defined by

$$
\begin{equation*}
u_{n}=3^{n}-2, \quad n \geq 1 . \tag{2}
\end{equation*}
$$

(a) Write down the first four terms of the sequence.

The same sequence can also be defined by the recurrence relation

$$
u_{n+1}=a u_{n}+b, \quad n \geq 1, \quad u_{1}=1,
$$

where $a$ and $b$ are constants.
(b) Find the values of $a$ and $b$.
5.


Figure 1
Figure 1 shows the curve with equation $y=8 x-x^{\frac{5}{2}}, x \geq 0$.
The curve meets the $x$-axis at the origin, $O$, and at the point $A$.
(a) Find the $x$-coordinate of $A$.
(b) Find the gradient of the tangent to the curve at $A$.
6.

$$
\mathrm{f}(x)=2 x^{2}-4 x+1
$$

(a) Find the values of the constants $a, b$ and $c$ such that

$$
\begin{equation*}
\mathrm{f}(x)=a(x+b)^{2}+c \tag{4}
\end{equation*}
$$

(b) State the equation of the line of symmetry of the curve $y=\mathrm{f}(x)$.
(c) Solve the equation $\mathrm{f}(x)=3$, giving your answers in exact form.
7. $\mathrm{f}(x) \equiv \frac{(x-4)^{2}}{2 x^{\frac{1}{2}}}, x>0$.
(a) Find the values of the constants $A, B$ and $C$ such that

$$
\begin{equation*}
\mathrm{f}(x)=A x^{\frac{3}{2}}+B x^{\frac{1}{2}}+C x^{-\frac{1}{2}} . \tag{3}
\end{equation*}
$$

(b) Show that

$$
\begin{equation*}
\mathrm{f}^{\prime}(x)=\frac{(3 x+4)(x-4)}{4 x^{\frac{3}{2}}} \tag{6}
\end{equation*}
$$

8. (a) Describe fully the single transformation that maps the graph of $y=\mathrm{f}(x)$ onto the graph of $y=\mathrm{f}(x-1)$.
(b) Showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes, sketch the graph of $y=\frac{1}{x-1}$.
(c) Find the $x$-coordinates of any points where the graph of $y=\frac{1}{x-1}$ intersects the graph of $y=2+\frac{1}{x}$. Give your answers in the form $a+b \sqrt{3}$, where $a$ and $b$ are rational.
9. A store begins to stock a new range of DVD players and achieves sales of $£ 1500$ of these products during the first month.

In a model it is assumed that sales will decrease by $£ x$ in each subsequent month, so that sales of $£(1500-x)$ and $£(1500-2 x)$ will be achieved in the second and third months respectively.

Given that sales total $£ 8100$ during the first six months, use the model to
(a) find the value of $x$,
(b) find the expected value of sales in the eighth month,
(c) show that the expected total of sales in pounds during the first $n$ months is given by $k n(51-n)$, where $k$ is an integer to be found.
(d) Explain why this model cannot be valid over a long period of time.
10. The curve $C$ with equation $y=\mathrm{f}(x)$ is such that

$$
\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{2}+4 x+k
$$

where $k$ is a constant.
Given that $C$ passes through the points $(0,-2)$ and $(2,18)$,
(a) show that $k=2$ and find an equation for $C$,
(b) show that the line with equation $y=x-2$ is a tangent to $C$ and find the coordinates of the point of contact.

## END

