OCR

A Level

A Level Maths

OCR Core Maths C3 June 2015
Model Solutions

Name:

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Total Marks:

June 15 OCR - C3

1. $y = \frac{5x+4}{3x-8}$ $\quad f = 5x+4 \quad g = 3x-8$

$f' = 5 \quad g' = 3$

$$\frac{dy}{dx} = \frac{5(3x-8) - 3(5x+4)}{(3x-8)^2}$$

at 2, $\frac{dy}{dx} = -13$

$$y + 7 = -13(x-2)$$

2i. $\cot\theta = 4$, $\tan\theta = 1/4$

$$\tan(\theta + 45°) = \frac{\tan\theta + \tan 45°}{1 - \tan\theta \tan 45}$$

$$= \frac{1/4 + 1}{1 - 1/4} = \frac{5}{3}$$

2ii. $\csc^2\theta = 1 + \cot^2\theta$

$\csc^2\theta = 1 + 4^2$

$\csc\theta = \sqrt{17}$

3. $V = 3(2+\sqrt{h})^6 - 192$

$\frac{dV}{dh} = 18.\frac{1}{2}h^{-1/2}(2+\sqrt{h})^5$

$= 9h^{-1/2}(2+h^{1/2})^5$

need $150 \div \frac{dV}{dh}$ (when $h = 1.4$)

$= 0.060337...$

$= 0.06$

4. $|x+3a| = 5a$

$x^2 + 6ax + 9a^2 = 25a^2$

$x^2 + 6ax - 16a^2 = 0$

$x = \frac{-6a \pm \sqrt{(6a)^2 - 4(1)(-16a^2)}}{2}$

$x = 2a$ or $-8a$

if $x = 2a$: $|x+7a| - |x-7a|$

$|9a| - |-5a| = 4a$

if $x = -8a$: $|x+7a| - |x-7a|$

$|-a| - |-15a|$

$a - 15a = -14a$

5i. $y = e^{3x} - 6e^{2x} + 32$

$y' = 3e^{3x} - 12e^{2x}$; $y'' = 9e^{3x} - 24e^{2x}$

at min. $\cancel{y'' > 0}$ $\Rightarrow$ $3e^{3x} = 12e^{2x}$ $(\div 3e^{2x})$
$y' = 0$ $e^{x} = 4$
$x = \ln 4$

at $x = \ln 4$, $y'' = 9e^{3\ln 4} - 24e^{2\ln 4}$
$= 576 - 384$
$= 192$
$192 > 0 \Rightarrow$ minimum

5ii. $\int_0^{\ln 4} e^{3x} - 6e^{2x} + 32$

$= \left[\frac{1}{3}e^{3x} - 3e^{2x} + 32x\right]_0^{\ln 4}$

$= \left(\frac{64}{3} - 48 + 32\ln 4\right) - \left(1/3 - 3\right)$

$= 32\ln 4 - 24$

$= 64\ln 2 - 24$

6ai. $y = 8\sin^{-1}(x - 3/2)$

at $(a, -4\pi)$
$$-4\pi = 8\sin^{-1}(a - 3/2)$$
$$-\tfrac{1}{2}\pi = \sin^{-1}(a - 3/2)$$
$$a - 3/2 = \sin(-\pi/2)$$
$$a = 1/2$$

at $(b, 4\pi)$
$$4\pi = 8\sin^{-1}(b - 3/2)$$
$$b - 3/2 = 1$$
$$b = 5/2$$

6aii. let $f(x) = 8\sin^{-1}(x - 3/2) - x$
$$f(1.7) = -0.0891\ldots$$
$$f(1.8) = 0.6375\ldots$$

change of sign $\Rightarrow$ $\alpha \in [1.7, 1.8]$

6b.
$$8\sin^{-1}(x - 3/2) = x$$
$$\sin^{-1}(x - 3/2) = \frac{x}{8}$$
$$x - 3/2 = \sin(x/8)$$
$$x_{n+1} = 3/2 + \sin(x_n/8)$$

$x_n = 1.7$
$$x_{n+1} = 1.710904323$$
$$= 1.712236508$$
$$= 1.712399234$$

$$= 1.712 \text{ to } 4\text{s.f.}$$

7i. $\int_1^9 (7x+1)^{1/3}\, dx$

$\frac{d}{dx}\, k(7x+1)^{4/3}$

$\frac{4}{3}k \cdot 7(7x+1)^{1/3}$

$\frac{28k}{3} = 1 \Rightarrow k = \frac{3}{28}$

$= \left[\frac{3}{28}(7x+1)^{4/3}\right]_1^9$

$= \frac{180}{7}$

7ii. $h = \frac{9-1}{2} = 4$

x		
1	y_0	2
5	y_1	$36^{1/3}$
9	y_2	4

$$\int \approx \frac{4}{3}\left[(2+4) + 4(36^{1/3}) + 0\right]$$

$$= 8 + \frac{16}{3} \cdot 36^{1/3}$$

$m = 8$

$n = \frac{16}{3}$

7iii. $\frac{180}{7} \approx 8 + \frac{16}{3} \cdot 36^{1/3}$

$36^{1/3} \approx \frac{93}{28}$

8i. $f(x) = 2 + \ln(x+3) \qquad x \geq 0$

$g(x) = ax^2 \qquad \forall x \in \mathbb{R}, \quad a > 0$

$f(e^4 - 3) = 6$

$gf(e^4 - 3) = g(6) = 36a = 9$

$a = 1/4$

8ii. $f^{-1}(x)$: let $y = 2 + \ln(x+3)$

$e^{y-2} = x + 3$

$x = e^{y-2} - 3$

$f^{-1}(x) = e^{x-2} - 3$

valid for $x \geq 0$ so $e^{x-2} \geq 3$

$x - 2 \geq \ln 3$

$x \geq 2 + \ln 3$

8iii. $ff(e^N - 3) = \ln(53e^2)$

$f(e^N - 3) = 2 + \ln(e^N - 3 + 3) = 2 + N$

$f(2+N) = \ln(53e^2)$

$f^{-1}[f(2+N)] = f^{-1}(\ln(53e^2))$

$2 + N = e^{\ln(53e^2) - 2} - 3$

$= 50$

$N = 48$

9i. $f(\theta) = \sin(\theta+30) + \cos(\theta+60)$

$= \sin\theta\cos30 + \cos\theta\sin30 + \cos\theta\cos60 - \sin\theta\sin60$

$= \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$= \cos\theta$

$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3$

LHS $\cos(4\theta) + 4\cos(2\theta)$

$\cos 2\theta \equiv 2\cos^2\theta - 1$

$\cos 4\theta \equiv 2\cos^2(2\theta) - 1$

$\cos^2(2\theta) = (2\cos^2\theta - 1)^2$

$2[(2\cos^2\theta - 1)^2] - 1 + 4(2\cos^2\theta - 1)$

$2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 + 8\cos^2\theta - 4$

$8\cos^4\theta \cancel{- 8\cos^2\theta} + 2 - 1 \cancel{+ 8\cos^2\theta} - 4$

$= 8\cos^4\theta - 3 = \text{RHS}$

9ii. $\frac{1}{f(4\theta) + 4f(2\theta) + 7} = \frac{1}{8\cos^4\theta + 4}$

$\cos^4\theta \in [1, 0]$

$\theta = 0$; $1/12$

$\theta = 90$; $1/4$

9iii. $8\cos^4(3\alpha) - 3 = 1$ $\quad 0 < \alpha < 60^\circ$

$0 < 3\alpha < 180^\circ$

$\cos^4(3\alpha) = 1/2$

$\cos(3\alpha) = \pm\sqrt[4]{1/2}$

$3\alpha = 32.765\ldots$ or $147.235\ldots$

$\alpha = 10.9^\circ$ $= 49.1^\circ$