OCR

A Level

A Level Maths

OCR Core Maths C2 June 2014 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

OCR June 14 C2

1i. Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(8)(10)\sin 65^\circ$

$= 36.3\text{ cm}^2$ (3 s.f.)

1ii. Cosine Rule : $BD^2 = 8^2 + 10^2 - 2(8)(10)\cos 65^\circ$

$BD = 9.82\text{ cm}$ (3 s.f.)

1iii. Sine Rule : $\frac{BC}{\sin 65^\circ} = \frac{8}{\sin 30^\circ}$

$BC = \frac{8\sin 65^\circ}{\sin 30^\circ}$

$= 14.5\text{ cm}$ (3 s.f.)

2i. $u_n = 3n - 1$

$u_1 = 3(1) - 1 = 2$

$u_2 = 3(2) - 1 = 5$

$u_3 = 3(3) - 1 = 8$

2ii. AP $\Rightarrow a = 2,\ d = 3$

$S_{40} = \frac{40}{2}\left\{2(2) + 39(3)\right\}$

$= 2420$

3i. Arc length $= 12 \times \frac{2\pi}{3}$

$= 8\pi$

3ii. Shaded Area = Area of Sector − Area of Δ

3ii. Sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2\left(\frac{2\pi}{3}\right)$

$= 48\pi$

Triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(12)^2\sin\left(2\pi/3\right)$

$= 36\sqrt{3}$

$\therefore$ Shaded Area $= 48\pi - 36\sqrt{3}$

4i. $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$

$\tan x\sin x - \cos x\tan x = 6\cos x \qquad (\div \cos x)$

$\tan^2 x - \tan x = 6 \qquad \left(\frac{\sin x}{\cos x} \equiv \tan x\right)$

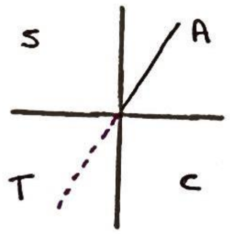
$\tan^2 x - \tan x - 6 = 0$

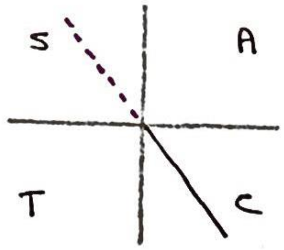
4ii. $(\tan x - 3)(\tan x + 2) = 0 \qquad 0 \leq x \leq 360°$

$\tan x = 3$ or $\tan x = -2$

P.V. $x = 71.6°$

P.V. $x = -63.4°$





$x = 71.6°, 251.6°$

-63.4 isn't in required range, but $-63.4° + 360°$ is.

$x = 116.6°, 296.6°$

$\therefore x = 71.6°, 117°, 252°, 297° \quad$ (3 s.f.)

5. $2^{4x-1} = 3^{5-2x}$

$\log(2^{4x-1}) = \log(3^{5-2x})$

$(4x-1)\log 2 = (5-2x)\log 3$

$4x\log 2 - \log 2 = 5\log 3 - 2x\log 3$

$4x\log 2 + 2x\log 3 = 5\log 3 + \log 2$

$x(4\log 2 + 2\log 3) = 5\log 3 + \log 2$

$$x = \frac{5\log 3 + \log 2}{4\log 2 + 2\log 3}$$

$5\log 3 + \log 2 = \log(3^5 \times 2) = \log 486$

$4\log 2 + 2\log 3 = \log(2^4 \times 3^2) = \log 144$

$$\therefore \quad x = \frac{\log 486}{\log 144}$$

6i. $$\left(x^3 + \frac{2}{x^2}\right)^4 = (x^3)^4 + {}^4C_1(x^3)^3.\left(\frac{2}{x^2}\right) + {}^4C_2(x^3)^2\left(\frac{2}{x^2}\right)^2 + {}^4C_3(x^3)\left(\frac{2}{x^2}\right)^3 + {}^4C_4\left(\frac{2}{x^2}\right)^4$$

$$= x^{12} + \frac{8x^9}{x^2} + \frac{24x^6}{x^4} + \frac{32x^3}{x^6} + \frac{16}{x^8}$$

$$= x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$$

6ii. $$\int \left(x^3 + \frac{2}{x^2}\right)^4 dx = \int x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}\, dx$$

$$= \frac{1}{13}x^{13} + x^8 + 8x^3 - 16x^{-2} - \frac{16}{7}x^{-7} + c$$

7i. $f(x) = 12 - 22x + 9x^2 - x^3$

$f(-2) = 12 - 22(-2) + 9(-2)^2 - (-2)^3$

$= 100$

7ii. $f(3) = 12 - 22(3) + 9(3)^2 - (3)^3$

$= 12 - 66 + 81 - 27$

$= 0 \quad \therefore \; (3-x)$ is a factor

7iii.

$$\begin{array}{r|l} & 4 - 6x + x^2 \\ \hline 3 - x & 12 - 22x + 9x^2 - x^3 \\ & \underline{12 - 4x} \\ & \quad -18x + 9x^2 \\ & \quad \underline{-18x + 6x^2} \\ & \qquad 3x^2 - x^3 \\ & \qquad \underline{3x^2 - x^3} \\ & \qquad\qquad 0 \end{array}$$

$\therefore \; f(x) = (3-x)(4 - 6x + x^2)$

7iv. $(3-x)(x^2 - 6x + 4) = 0$

$x = 3$ or $x^2 - 6x + 4 = 0$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(4)}}{2}$$

$= 3 \pm \sqrt{5}$

8a GP, $a = 50$ $\quad r = 0.8$

$u_k = ar^{k-1} = 50 . (0.8)^{k-1}$

$\therefore \quad 50 \times 0.8^{k-1} < 0.15 \quad (\div 50)$

$0.8^{k-1} < 0.003$

$\log(0.8^{k-1}) < \log 0.003$

8a. $(k-1)\log 0.8 < \log 0.003$

$k-1 > \dfrac{\log 0.003}{\log 0.8}$ ($\log 0.8 < 0$ $\therefore$ inequality sign flips)

$k > 1 + \dfrac{\log 0.003}{\log 0.8}$

$k > 27.03$

$\therefore \ k = 28$

8b. GP $u_2 = -3$, $S_\infty = 4$

$ar = -3$

$a = -3/r$ ① $\dfrac{a}{1-r} = 4$ ②

'Sub ① into ②'

$\dfrac{-3/r}{1-r} = 4$

$-\dfrac{3}{r} = 4(1-r)$

$-3/r = 4 - 4r$ $(\times r)$

$-3 = 4r - 4r^2$

$4r^2 - 4r - 3 = 0$

$(2r+1)(2r-3) = 0$

$r = -1/2$ or $r = 3/2$

Since S_∞ only valid for $|r| < 1$, $r \neq 3/2$

$\therefore \ r = -1/2$

'Sub $r = -1/2$ into ①'

$a = \dfrac{-3}{(-1/2)}$

$= 6$

9i. $y = -3 + 2\sqrt{x+4}$ $\qquad h = \frac{5-0}{2} = 2.5$

x	y
0	1
2.5	$-3+2\sqrt{6.5}$
5	3

$$\text{Area} \approx \frac{1}{2} \times 2.5 \left\{ (1+3) + 2(-3+2\sqrt{6.5}) \right\}$$

$$= 10.2 \quad (3sf)$$

9ii. $B = \text{Rectangle} - A$

$$= (3\times 5) - 10.2$$

$$= 4.8$$

9iii. $y = -3 + 2\sqrt{x+4}$

$$(y+3) = 2\sqrt{x+4}$$

$$\frac{y+3}{2} = \sqrt{x+4}$$

$$\frac{(y+3)^2}{4} = x+4$$

$$x = \frac{(y+3)^2}{4} - 4$$

$$= \frac{1}{4}y^2 + \frac{1}{4}6y + \frac{9}{4} - 4$$

$$= \frac{1}{4}y^2 + \frac{3}{2}y - 7/4$$

when $x = 0$, $y = 1$

$$\therefore \int_1^3 \frac{y^2}{4} + \frac{3y}{2} - 7/4 \; dy$$

$$= \left[\frac{y^3}{12} + \frac{3y^2}{4} - 7/4\, y \right]_1^3$$

9iii

$$= \left(\frac{27}{12} + \frac{27}{4} - \frac{21}{4}\right) - \left(\frac{1}{12} + \frac{3}{4} - \frac{7}{4}\right)$$

$$= \frac{14}{3}$$