OCR

A Level

A Level Maths

OCR Core Maths C1 June 2014 Model Solutions

Name:

M M E

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Total Marks:

OCR June 14 C1

1. $5x^2 + 10x + 2$

$5[x^2 + 2x] + 2$

$5[(x+1)^2 - 1] + 2$

$5(x+1)^2 - 5 + 2$

$5(x+1)^2 - 3$

2i. $\frac{6}{\sqrt{3}} \quad (\times \sqrt{3}) \quad \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

2ii. $10\sqrt{3} - 6\sqrt{27}$

$= 10\sqrt{3} - 6\sqrt{9 \times 3}$

$= 10\sqrt{3} - 18\sqrt{3}$

$= -8\sqrt{3}$

2iii. $3^{5/2} = (3^{1/2})^5 = (\sqrt{3})^5$

$= \underbrace{\sqrt{3} \times \sqrt{3}}_{3} \times \underbrace{\sqrt{3} \times \sqrt{3}}_{3} \times \sqrt{3}$

$= 9\sqrt{3}$

3. $4x^4 + 3x^2 - 1 = 0$ Let $y = x^2$, $y^2 = x^4$

$4y^2 + 3y - 1 = 0$

$(4y - 1)(y + 1) = 0$

$y = 1/4$ or $y = -1$

$y = 1/4 \Rightarrow x^2 = 1/4 \; ; \; x = \pm 1/2$

$y = -1 \Rightarrow x^2 = -1 \; ; \; x = \sqrt{-1}$ ×

4i. $P(2,5)$

$f(x) \rightarrow f(x)+2$ translation 2 units up

$\therefore (2,7)$ (add 2 to y value)

4ii. $f(x) \rightarrow f(2x)$ stretch s.f. $\frac{1}{2}$ in x direction

$\therefore (1,5)$ (half the x values)

4iii. $f(x) \rightarrow f(x+4)$ translation 4 units in the negative x direction

5i. $5 < 6x+3 < 14$ (-3)

$2 < 6x < 11$ $(\div 6)$

$\frac{1}{3} < x < \frac{11}{6}$

5ii. $x(3x-13) \geqslant 10$

$3x^2 - 13x \geqslant 10$

$3x^2 - 13x - 10 \geqslant 0$

$(3x+2)(x-5) \geqslant 0$

C.V.s $x = 5$, $x = -\frac{2}{3}$

$\therefore x \leqslant -\frac{2}{3}$

$x \geqslant 5$

6i. $y = 6x^3 + \frac{4}{\sqrt{x}} + 5x$

$= 6x^3 + 4x^{-1/2} + 5x$

$\frac{dy}{dx} = 18x^2 - 2x^{-3/2} + 5$

6ii. $\frac{d^2y}{dx^2} = 36x + 3x^{-5/2}$

7i. A (5,7) B (-1,-5)

Midpoint $= \left(\frac{5+-1}{2}, \frac{7+-5}{2}\right) = (2,1)$

7ii. grad AB $= \frac{7--5}{5--1} = \frac{12}{6} = 2$

line is $\perp$ $\therefore$ grad of line $= -1/2$

A is on the line $\therefore$

$y - 7 = -\frac{1}{2}(x-5) \quad \times 2$

$2y - 14 = -x + 5$

$2y + x - 19 = 0$

8i. $y = 3x^3 - 7x + \frac{2}{x}$

$= 3x^3 - 7x + 2x^{-1}$

$\frac{dy}{dx} = 9x^2 - 7 - 2x^{-2}$

at stat. point $\frac{dy}{dx} = 0$

when $x = 1$, $\frac{dy}{dx} = 9(1)^2 - 7 - \frac{2}{1^2}$

$= 9 - 7 - 2$

$= 0$

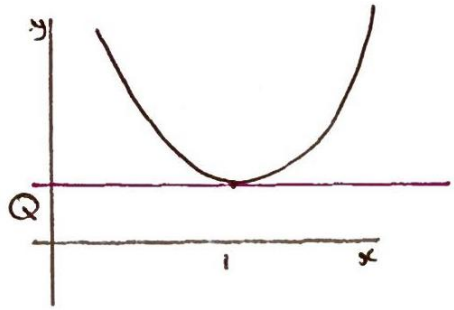
$\therefore$ stat. when $x = 1$

8ii. $\frac{d^2y}{dx^2} = 18x + 4x^{-3}$

when $x = 1$; $\frac{d^2y}{dx^2} = 18(1) + \frac{4}{(1)^3} = 18 + 4 = 22$

$22 > 0$ $\therefore$ minimum point

8iii. Tangent to a stat. point is a horizontal line



∴ to find the y coord. of Q, we need the coordinates of the minimum.

$x = 1$; $y = 3(1)^3 - 7(1) + \frac{2}{1}$

$= 3 - 7 + 2$

$= -2$

∴ Q $(0, -\frac{2}{5})$

9i. $(x-2)^2 + (y+5)^2 = 25$

Centre : $(2, -5)$

Radius = $\sqrt{25} = 5$

The radius is 5 ∴ the highest point of the circle would be at $(2,0)$, which touches the x axis.

9ii. $P(6,k)$

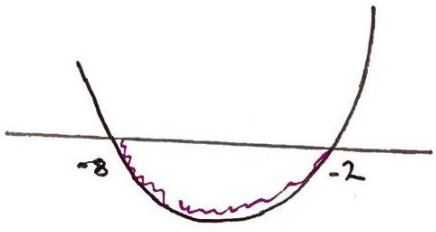
If $(6,k)$ is inside the circle $(6-2)^2 + (k+5)^2 < 25$

$16 + (k+5)^2 < 25$

$16 + k^2 + 10k + 25 < 25$

$k^2 + 10k + 16 < 0$

$(k+2)(k+8) < 0$



c.v.s $k = -2$, $k = -8$

9iii. $2y = x$ ①

$(x-2)^2 + (y+5)^2 = 25$ ②

'sub ① into ②'

$(2y-2)^2 + (y+5)^2 = 25$

$4y^2 - 8y + 4 + y^2 + 10y + 25 = 25$

$5y^2 + 2y + 4 = 0$

disc. $= 2^2 - 4(5)(4)$

$= 4 - 80$

$= -76$

disc < 0 $\therefore$ no real solutions

$\therefore$ line does not meet the circle

10i. $y = (x+2)^2(2x-3)$

root at $x = 3/2$

double root at $x = -2$

$x = 0$, $y = 2^2(-3) = -12$ so crosses y axis at -12