OCR

A Level

A Level Maths

OCR Core Maths C2 June 2013 Model Solutions

Name:

MME
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Total Marks:

OCR June 13 C2

1. $\int_5^{11} \frac{8}{x}\,dx$ 3 strips $\Rightarrow$ $h = \frac{11-5}{3} = 2$

x	y
5	$8/5$
7	$8/7$
9	$8/9$
11	$8/11$

Area $= \frac{1}{2} \times 2 \left\{ \left(8/5 + 8/11 \right) + 2\left(8/7 + 8/9 \right) \right\}$

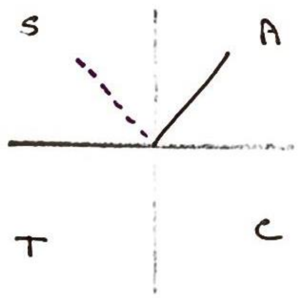
$= 6.39$ (3 s.f.)

2i. $\sin \frac{1}{2}x = 0.8$ $0^\circ \leq x \leq 360^\circ$

let $\phi = \frac{1}{2}x$ $0^\circ \leq \phi \leq 180^\circ$

$\sin \phi = 0.8$

P.V. 53.1°



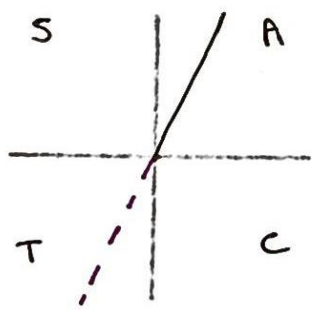
$\phi = 53.1^\circ, 126.9^\circ$

$x = 106^\circ, 254^\circ$ (3 s.f.)

2ii. $\sin x = 3\cos x$ $(\div \cos x)$

$\tan x = 3$

P.V. 71.6°



$x = 71.6^\circ, 252^\circ$ (3 s.f.)

3i. $(2+5x)^6 = 2^6 + {}^6C_1 2^5 (5x) + {}^6C_2 2^4 (5x)^2 + \dots$

$= 64 + 960x + 6000x^2 + \dots$

3ii. $(3+cx)^2(2+5x)^6$ $\quad x$ coefficient $= 4416$

$(3+cx)^2 = 9 + 6cx + \dots$ (don't need x^2 terms)

$(9 + 6cx + \dots)(64 + 960x + \dots)$

x terms : $9 \times 960x = 8640x$

$6cx \times 64 = 384cx$

$= x(8640 + 384c) = 4416x$

$\therefore \quad 384c + 8640 = 4416$

$384c = -4224$

$c = -11$

4a. $\int 5x^3 - 6x + 1 \, dx$

$= \frac{5}{4}x^4 - 3x^2 + x + c$

4bi. $\int 24x^{-3} \, dx = -\frac{12}{x^2} + c$

4bii. $\int_a^\infty 24x^{-3} \, dx = 3$

$\therefore \lim_{b \to \infty} \left[-\frac{12}{x^2} \right]_a^b = 3$

$-\frac{12}{b^2} - -\left(\frac{12}{a^2}\right) = 3$

as $b \to \infty$, $-\frac{12}{b^2} \to 0$

$\therefore \quad \frac{12}{a^2} = 3$, $\quad 12 = 3a^2 \Rightarrow a = 2 \quad (a > 0)$

5i. Area BDC = Sector ABC − $\triangle$ ABD

Sector : $\frac{1}{2}r^2\theta$

$= \frac{1}{2}(16)^2(0.8) = 102.4$

$\triangle$: $\frac{1}{2}ab\sin C$

$= \frac{1}{2}(16)(7)\sin(0.8) = 40.1719\ldots$

BDC $= 102.4 - 40.1719\ldots$

$= 62.2\ \text{cm}^2$ (3 s.f.)

5ii. BC = arc length = $r\theta$

$= 16(0.8) = 12.8$

DC $= 16 - 7 = 9$

BD : Cosine Rule : $BD^2 = 16^2 + 7^2 - 2(16)(7)\cos(0.8)$

$BD = 12.204\ldots$

Perimeter = BC + DC + BD = 34.0 cm (3 s.f.)

6i. $u_1 = 6$, $u_2 = 7.8$

AP $\Rightarrow$ $a = 6$, $d = u_2 - u_1 = 1.8$

$u_{30} = a + (30-1)d$

$= 6 + 29(1.8)$

$= 58.2$

6ii. G.P. $\Rightarrow$ $r = \frac{u_2}{u_1} = \frac{7.8}{6} = 1.3$

$S_N \leqslant 1800$

$\frac{a(1-r^N)}{1-r} \leqslant 1800$

6ii.

$$\frac{6(1-1.3^N)}{1-1.3} \leq 1800$$

$$-20(1-1.3^N) \leq 1800$$

NB. Inequality flips when we multiply / divide by a negative number.

$$1-1.3^N \geq -90$$

$$-1.3^N \geq -91$$

$$1.3^N \leq 91$$

$$\log(1.3^N) \leq \log 91$$

$$N \log 1.3 \leq \log 91$$

$$N \leq \frac{\log 91}{\log 1.3}$$

$$N \leq 17.2\ldots$$

$$\therefore \quad N = 17$$

7i.

$$\int_1^4 x^{3/2} - 1 \; dx$$

$$\left[\frac{2}{5}x^{5/2} - x\right]_1^4$$

$$F[4] = \frac{2}{5}(4)^{5/2} - 4 = 44/5$$

$$F[1] = \frac{2}{5}(1)^{5/2} - 1 = -3/5$$

$$\int = \frac{44}{5} - (-3/5)$$

$$= 47/5 \qquad (= 9\,2/5)$$

7ii.

Shaded area = Area under curve − Area of triangle

To find the area of the triangle, we need to know where the tangent intersects the x axis.

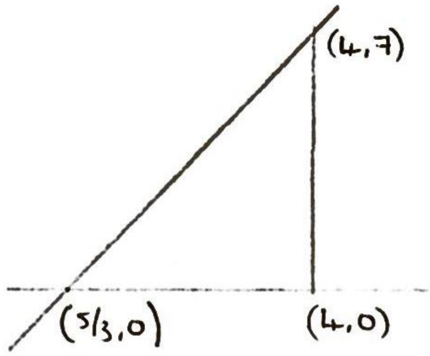
$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}, \quad \text{at } (4, 7) \quad \frac{dy}{dx} = \frac{3}{2}(4)^{1/2} = 3$$

Tangent: $y - 7 = 3(x-4)$

7ii. When $y = 0$: $0 - 7 = 3(x - 4)$

$-7 = 3x - 12$

$x = 5/3$



$$\text{Area} = \frac{1}{2}(b \times h)$$

$$= \frac{1}{2}\left\{(4 - 5/3) \times 7\right\}$$

$$= 49/6$$

$\therefore$ Shaded Area $= 47/5 - 49/6$

$= 37/30$

8ia. $x = 0$; $y = a^0 = 1 \Rightarrow (0,1)$

8ib. $x = 0$; $y = 4(b)^0 = 4 \Rightarrow (0,4)$

8ic. from diagram $y = a^x$ is an increasing function

$\therefore$ $a > 1$, e.g. $a = 2$

$y = 4b^x$ is a decreasing function

$\therefore$ $0 < b < 1$, e.g. $b = 1/2$

8c. $y = a^x$
$y = 4b^x$ $\therefore$ $a^x = 4b^x$ ①

$ab = 2 \Rightarrow b = \frac{2}{a}$

'Sub $b = \frac{2}{a}$ into ①'

$a^x = 4\left(\frac{2^x}{a^x}\right)$ $(\times a^x)$

$a^x \times a^x = 2^2 \times 2^x$

$a^{2x} = 2^{x+2}$

8ii. $\log_2 a^{2x} = \log_2 2^{x+2}$

$(2x)\log_2 a = x + 2$

$x(2\log_2 a - 1) = 2$

$$x = \frac{2}{2\log_2 a - 1}$$

9i. $f(x) = 4x^3 - 7x - 3$

$f(2) = 4(2)^3 - 7(2) - 3$

$= 15$

9ii. $f(-1/2) = 4(-\frac{1}{2})^3 - 7(-\frac{1}{2}) - 3$

$= 0$ $\therefore$ $(2x+1)$ is a factor

$$\begin{array}{r|l} & 2x^2 - x - 3 \\ \hline 2x+1 & 4x^3 + 0x^2 - 7x - 3 \\ & \underline{4x^3 + 2x^2} \\ & \quad -2x^2 - 7x \\ & \quad \underline{-2x^2 - x} \\ & \qquad -6x - 3 \\ & \qquad \underline{-6x - 3} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ $f(x) = (2x+1)(2x^2 - x - 3)$

$= (2x+1)(2x-3)(x+1)$

9iii. $4\cos^3\theta - 7\cos\theta - 3 = 0$

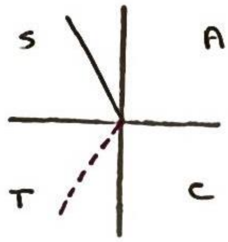
$(x = \cos\theta)$

$\therefore$ $(2\cos\theta + 1)(2\cos\theta - 3)(\cos\theta + 1) = 0$

9iii. $2\cos\theta + 1 = 0 \qquad 0 \leq \theta \leq 2\pi$

$\cos\theta = -\frac{1}{2}$

P.V. $= \frac{2\pi}{3}$



$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

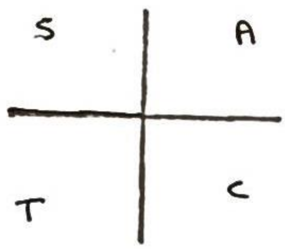
$2\cos\theta - 3 = 0$

$\cos\theta = \frac{3}{2}$ X

$\cos\theta + 1 = 0$

$\cos\theta = -1$

P.V. π



$\therefore \theta = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$