OCR

A Level

A Level Maths

OCR Core Maths C1 June 2013
Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

OCR - June 13 C1

1i. $4\sqrt{15} \times \sqrt{3}$ $\qquad \sqrt{15} = \sqrt{3\times5}$

$= 4 \times \sqrt{5} \times \sqrt{3} \times \sqrt{3}$

$= 4\sqrt{5} \times 3$

$= 12\sqrt{5}$

1ii. $\frac{20}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$

1iii. $5^{3/2} = 5^1 \times 5^{1/2} = 5\sqrt{5}$

2. $8x^6 + 7x^3 - 1 = 0$ $\qquad$ Let $y = x^3$, $y^2 = x^6$

$8y^2 + 7y - 1 = 0$

$(8y - 1)(y + 1) = 0$

$y = -1$ or $y = 1/8$

$y = -1$; $x^3 = -1 \Rightarrow x = -1$

$y = 1/8$; $x^3 = \frac{1}{8} \Rightarrow x = 1/2$

3i. $F(x) = \frac{6}{x^2} + 2x$

$= 6x^{-2} + 2x$

$f'(x) = -12x^{-3} + 2$

3ii. $f''(x) = 36x^{-4}$

4i. $3x^2 + 9x + 10$

$3(x^2+3x) + 10$

$3\left[(x+3/2)^2 - (3/2)^2\right] + 10$

$3(x+3/2)^2 - 3\left(\frac{9}{4}\right) + 10$

$3(x+3/2)^2 - \frac{27}{4} + \frac{40}{4}$

$3(x+3/2)^2 + 13/4$

4ii. Minimum at $(-3/2, 13/4)$

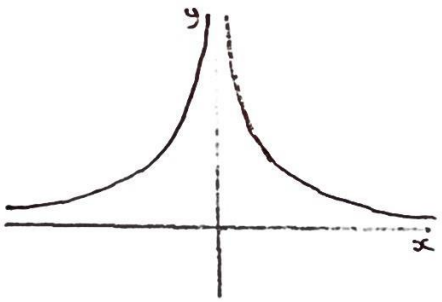
4iii. $3x^2 + 9x + 10$

$\therefore$ disc $= (9)^2 - 4(3)(10)$

$= 81 - 120$

$= -39$

5i. $y = \frac{2}{x^2}$



5ii. $\frac{2}{x^2} \rightarrow \frac{2}{(x+5)^2}$ translation 5 units in negative x direction

5iii. $y = \frac{2}{x^2} \rightarrow y = \frac{1}{x^2}$; let $f(x) = \frac{2}{x^2}$

$f(x) \rightarrow \frac{1}{2} f(x)$ $\therefore$ stretch s.f. 1/2 in y direction

6i. $x^2 + y^2 + 8y - 24 = 0$

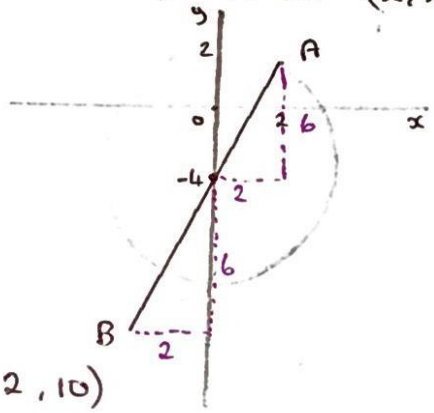
$(x-0)^2 + (y+4)^2 - 16 - 24 = 0$

$(x-0)^2 + (y+4)^2 = 40$

$\therefore$ centre $(0, -4)$

radius $= \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$

6ii. AB is a diameter, A at $(2, 2)$



$\therefore$ B at $(-2, 10)$

7i. $3 - 8x > 4 \quad (-3)$

$-8x > 1 \quad (\times -1)$

$8x < -1 \quad (\div 8)$

$x < -\frac{1}{8}$

7ii. $(2x-4)(x-3) \leq 12$

$2x^2 - 10x + 12 \leq 12 \quad (-12)$

$2x^2 - 10x \leq 0$

$2x(x-5) \leq 0$

C.V.s $x = 0 \quad x = 5$

$0 \leq x \leq 5$

8. A $(-2, 6)$ B $(3, -8)$

Midpoint AB $= \left(\frac{-2+3}{2}, \frac{6+-8}{2}\right) = \left(\frac{1}{2}, -1\right)$

$x - 3y + 15 = 0$

$3y = x + 15$

$y = \frac{1}{3}x + 5$ $\therefore$ grad $= 1/3$

l is $\perp$ $\therefore$ grad of $l = -3$

$y + 1 = -3\left(x - \frac{1}{2}\right)$

$y + 1 = -3x + \frac{3}{2}$ $(\times 2)$

$2y + 2 = 3 - 6x$

$2y + 6x - 1 = 0$

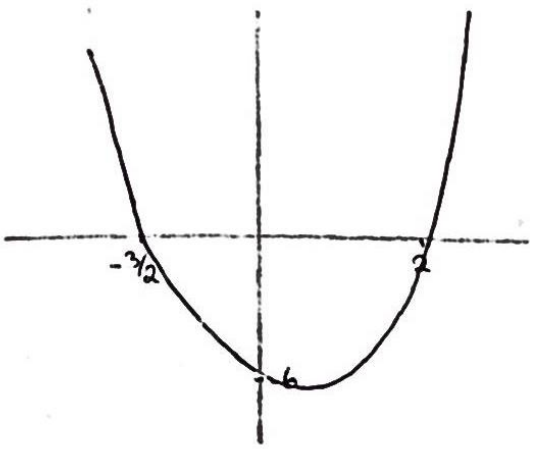
9. $y = 2x^2 - x - 6$

when $x = 0$ $y = -6$ $\therefore$ crosses y axis at -6

$y = (2x + 3)(x - 2)$

so roots at $2, -3/2$

Positive x^2 $\therefore$ $\cup$ shaped



9ii. $y = 2x^2 - x - 6$

$\frac{dy}{dx} = 4x - 1$

function is decreasing when $\frac{dy}{dx} < 0$

$\therefore \quad 4x - 1 < 0$

$4x < 1$

$x < 1/4$

9iii. $y = 2x^2 - x - 6 \quad ①$

$y = 4 \quad ②$

'Sub ② into ①'

$2x^2 - x - 6 = 4$

$2x^2 - x - 10 = 0$

$(2x - 5)(x + 2) = 0$

$x = -2$ or $x = 5/2$ - x coordinates of P & Q

y coordinates are 4

P $(-2, 4)$ Q $(5/2, 4)$

$\therefore \quad PQ = 2 + 5/2 = 9/2$ (differences in x's)

10i. $y = (1 - x)(x^2 + 4x + k)$

$y = x^2 + 4x + k - x^3 - 4x^2 - kx$

$= -x^3 - 3x^2 + 4x - kx + k$

$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$

Stat. pt when $x = -3$ $\therefore \frac{dy}{dx} = 0$ when $x = -3$

$0 = -3(-3)^2 - 6(-3) + 4 - k$

$k = -27 + 18 + 4$

$k = -5$

10ii. $\frac{d^2y}{dx^2} = -6x - 6$

when $x = -3$, $\frac{d^2y}{dx^2} = -6(-3) - 6$

$= 18 - 6$

$= 12$

$\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum pt when $x = -3$

10iii. $y = 9x - 9$ $\therefore$ grad of tangent $= 9$

We need to find the points on the curve where the gradient is 9.

$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$ $(k = -5)$

$= -3x^2 - 6x + 9$

To find A set $\frac{dy}{dx} = 9$

$9 = -3x^2 - 6x + 9$

$3x^2 + 6x = 0$

$3x(x + 2) = 0$

$x = 0$ or $x = -2$

$x = 0$; y coord of tangent $= 9(0) - 9 = -9$

y coord. of curve $= 1 \times -5 = -5$

$x = -2$ y coord. of tangent $: 9(-2) - 9 = -27$

y coord. of curve $: (1 - -2)((-2)^2 + 4(-2) - 5)$

$= 3(4 - 8 - 5)$

$= -27$

$\therefore$ A at $(-2, -27)$