

OCR

A Level

A Level Maths

OCR Core Maths C2 June 2012
Model Solutions

Name:

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Mathsmadeeasy.co.uk

Total Marks:

OCR Jun 12 C2

$$\begin{aligned} \text{1i. } (3+2x)^5 &= 3^5 + {}^5C_1 3^4 (2x) + {}^5C_2 3^3 (2x)^2 \\ &\quad + {}^5C_3 3^2 (2x)^3 + {}^5C_4 3 (2x)^4 + (2x)^5 \\ &= 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5 \end{aligned}$$

$$\text{1ii. } (3-2x)^5 = 243 - 810x + 1080x^2 - 720x^3 + 240x^4 - 32x^5$$

$$(3+2x)^5 + (3-2x)^5 = 486 + 2160x^2 + 480x^4$$

$$\text{2i. } \int x^2 - 2x + 5 \, dx = \frac{1}{3}x^3 - x^2 + 5x + c$$

$$y = \frac{1}{3}x^3 - x^2 + 5x + c$$

$$11 = \frac{1}{3}(3)^3 - 3^2 + 5(3) + c$$

$$11 = 9 - 9 + 15 + c, \quad c = -4$$

$$y = \frac{1}{3}x^3 - x^2 + 5x - 4$$

$$3: \quad \pi = 180^\circ$$

$$\frac{2\pi}{5} = 72^\circ$$

$$3: \quad A = \frac{1}{2} r^2 \theta$$

$$45\pi^2 = \frac{1}{2} r^2 \cdot \frac{2\pi}{5} \quad \Rightarrow \quad r = 15 \text{ cm}$$

$$3: \quad A \text{ of } \Delta = \frac{1}{2} (15)(15) \sin\left(\frac{2\pi}{5}\right)$$

$$= 106.99$$

$$A \text{ of seg. } 45\pi - \Delta = 34.4 \text{ cm}^2 \quad (3 \text{ s.f.})$$

$$4: \quad 4\cos^2 x + 7\sin x - 7 = 0$$

use $\cos^2 x = 1 - \sin^2 x$

$$4 - 4\sin^2 x + 7\sin x - 7 = 0$$

$$4\sin^2 x - 7\sin x + 3 = 0$$

$$(4\sin x - 3)(\sin x - 1) = 0$$

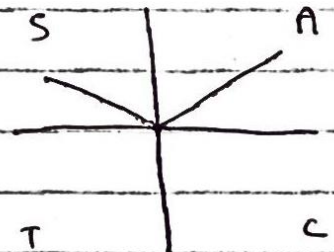
$$\sin x = 1$$

$$\sin x = \frac{3}{4}$$

$$x = 90^\circ$$

$$x = 48.59^\circ$$

$$131.41^\circ$$



5a. $u_{n+1} = 2/u_n$

i) $u_1 = 4, u_2 = 1/2, u_3 = 4$

ii) periodic / oscillating

5b. $u_9 = a + 8d = 18$

$$S_9 = \frac{9(a+18)}{2} = 72$$

$$9a + 162 = 144$$

$$9a = -18$$

$$a = -2$$

$$18 = -2 + 8d$$

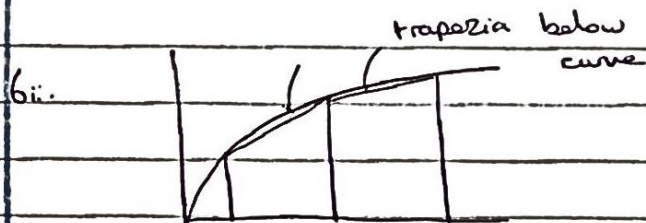
$$8d = 20 \Rightarrow d = 5/2$$

6. $\int_1^9 4\sqrt{x} dx$

x	f(x)
$y_0 = 1$	4
$y_1 = 5$	$4\sqrt{5}$
$y_2 = 9$	12

$h = \frac{9-1}{2} = 4$

$$\int \approx \frac{1}{2}(4) \left\{ (4+12) + 8\sqrt{5} \right\} = 2(16+8\sqrt{5}) = 32+16\sqrt{5}$$

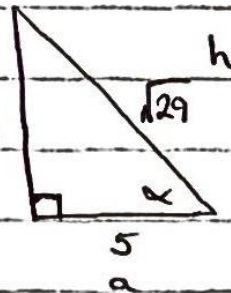


6iii. $4 \int_1^9 x^{1/2} dx = \left[\frac{8}{3} x^{3/2} \right]_1^9$

$$= \frac{72 - 8}{3}$$

$$= \frac{202}{3}$$

7ai. $\tan \alpha = \frac{2}{5} = 2$



$$\cos \alpha = \frac{5}{\sqrt{29}}$$

7ii. $\sin \beta = \frac{3}{7}$; $\sin^2 \beta = \frac{9}{49}$

use $\sin^2 \beta = 1 - \cos^2 \beta$

$$1 - \cos^2 \beta = \frac{9}{49}$$

$$\cos^2 \beta = \frac{40}{49}$$

$$\cos \beta = \sqrt{\frac{40}{49}} = \frac{\sqrt{40}}{7}$$

7b. $\frac{\sin y}{6} = \frac{\sin 60}{8}$

$$\sin y = \frac{6}{8} \cdot \sin 60$$

$$= \frac{3\sqrt{3}}{8}$$

$$8i. \quad f(x) = x^3 + (a-3)x + 2b$$

$$g(x) = 3x^3 + x^2 + 5ax + 4b$$

$$f(2) = 2^3 + 2(a-3) + 2b$$

$$0 = 2 + 2a + 2b \Rightarrow a + b = -1 \quad \textcircled{1}$$

$$g(2) = 3(2)^3 + 2^2 + 10a + 4b$$

$$0 = 28 + 10a + 4b \Rightarrow 5a + 2b = -14 \quad \textcircled{2}$$

$$2a + 2b = -2 \quad \textcircled{1} \times 2$$

$$3a = -12$$

$$a = -4, \quad b = 3$$

$$8ii. \quad f(x) = x^3 - 7x + 6$$

$$f(1) = 0$$

$$= (x-2)(x-1)(x+3)$$

$$g(x) = 3x^3 + x^2 - 20x + 12$$

$$g(-3) = 0 \Rightarrow x+3 \text{ a factor}$$

$$g(x) = (x-2)(x+3)(3x-2)$$

hence common factor of $(x+3)$

$$9a: \quad u_n = \log_2 27 + (n-1) \log_2 x$$

$$u_n = \log_2 27 + 3 \log_2 x$$

$$= \log_2 27 + \log_2 x^3$$

$$= \log_2 (27x^3)$$

$$9a: \quad 6 = \log_2 (27x^3)$$

$$64 = 27x^3 \quad \Rightarrow \quad x = \frac{4}{3}$$

$$9b: \quad a = \log_2 27 \quad r = \log_2 y$$

S_∞ valid for $|\log_2 y| < 1$

$$\log_2 y < 1$$

$$-\log_2 y < 1$$

$$y < 2$$

$$\log_2 y > -1$$

$$y > \frac{1}{2}$$

$$\frac{1}{2} < y < 2$$

$$9b: \quad S_\infty = 3 = \frac{\log_2 27}{1 - \log_2 y}$$

$$3 - 3 \log_2 y = \log_2 27$$

$$3 = \log_2 27y^3$$

$$8 = 27y^3$$

$$y = \frac{2}{3}$$