OCR

A Level

A Level Maths

OCR Core Maths C1 June 2012
Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

OCR · June 12 C1

1. $(x-5)(x^2+3) - (x+4)(x-1)$

$= x^3 + 3x - 5x^2 - 15 - (x^2 + 3x - 4)$

$= x^3 - 6x^2 - 11$

2i. $\sqrt[4]{7} = 7^{1/4}$

2ii. $\frac{1}{7\sqrt{7}} = \frac{1}{7^1 \cdot 7^{1/2}} = \frac{1}{7^{3/2}} = 7^{-3/2}$

2iii. $7^4 \times 49^{10} = 7^4 \times (7^2)^{10} = 7^4 \times 7^{20} = 7^{24}$

3i. $3x - 5y - 20 = 0$

$5y = 3x - 20$

$y = \frac{3}{5}x - 4 \quad \therefore$ gradient $= 3/5$

3ii. at P $y = 0 \quad \therefore \quad 3x - 20 = 0$

$3x = 20$

$x = \frac{20}{3} \; ; \; (\frac{20}{3}, 0)$

at Q $x = 0 \quad \therefore \quad -5y - 20 = 0$

$5y = -20$

$y = -4 \quad (0, -4)$

Midpoint $= \left(\frac{20/3 + 0}{2}, \frac{0 + (-4)}{2} \right)$

$= \left(\frac{10}{3}, -2 \right)$

4i. $2x^2 - 20x + 49$

$= 2\left[x^2 - 10x\right] + 49$

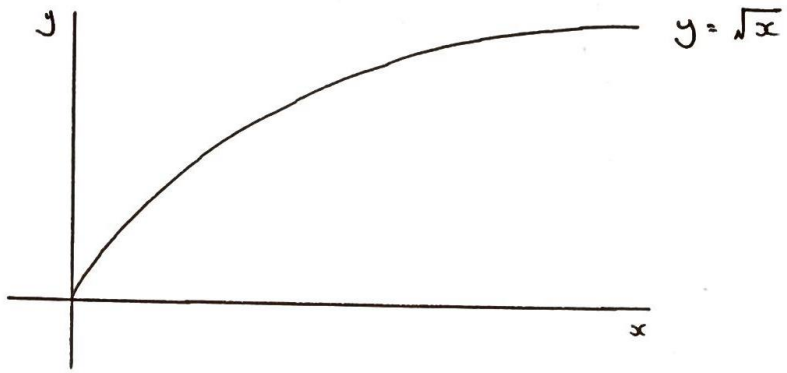
$= 2\left[(x-5)^2 - 25\right] + 49$

$= 2(x-5)^2 - 2(25) + 49$

$= 2(x-5)^2 - 1$ $\quad \therefore \quad p = 2,\ q = 5,\ r = -1$

4ii. Vertex at $(5, -1)$

5i.



5ii. $\sqrt{x} \longrightarrow \sqrt{x-4}$ translation 4 units in the positive x direction

5iii. $\sqrt{x} \longrightarrow \sqrt{\frac{x}{5}}$ Since $f(x) \to f(ax)$ stretch s.f $\frac{1}{a}$ in x direction

6. $y = \frac{6}{x^2} - 5$

$y = 6x^{-2} - 5$

$\frac{dy}{dx} = -12x^{-3}$

at $x = 2$, $\frac{dy}{dx} = -\frac{12}{(2)^3} = -\frac{12}{8} = -\frac{3}{2}$

$\therefore$ m of normal $= 2/3$

$y + \frac{7}{2} = \frac{2}{3}(x-2)$ $\times 6$

$6y + 21 = 4x - 8$

$4x - 6y - 21 = 0$

When $x = 2$, $y = \frac{6}{2^2} - 5 = -7/2$

so point at $(2, -7/2)$

7. $x - 6x^{\frac{1}{2}} + 2 = 0$ Let $y = x^{1/2}$

$y^2 = x$

$y^2 - 6y + 2 = 0$

$(y-3)^2 - 9 + 2 = 0$

$(y-3)^2 = 7$

$y = 3 \pm \sqrt{7}$

$x = (3 \pm \sqrt{7})^2$

$(3+\sqrt{7})(3+\sqrt{7}) = 9 + 6\sqrt{7} + 7 \; ; \quad 16 + 6\sqrt{7}$

$(3-\sqrt{7})(3-\sqrt{7}) = 9 - 6\sqrt{7} + 7 \; ; \quad 16 - 6\sqrt{7}$

$\therefore \quad x = 16 \pm 6\sqrt{7}$, $p = 16$, $q = 6$, $r = 7$

8i. $y = x^4 + 32x$

$\frac{dy}{dx} = 4x^3 + 32$ at stat. points $\frac{dy}{dx} = 0$

$4x^3 + 32 = 0$

$4x^3 = -32$

$x^3 = -8$

$x = -2$

When $x = -2$, $y = (-2)^4 + 32(-2)$

$= 16 - 64$

$= -48 \quad \therefore \quad (-2, -48)$

8ii. $\frac{d^2y}{dx^2} = 12x^2$

when $x = -2$, $\frac{d^2y}{dx^2} = 12(-2)^2 = 12(4) = 48$

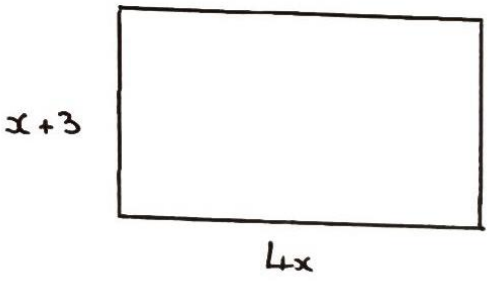
$48 > 0 \quad \therefore$ minimum point at $(-2, -48)$

8:: Increasing when $\frac{dy}{dx} > 0$

$4x^3 + 32 > 0$

$\therefore \quad x > -2 \quad$ (from prev. part)

9:



Area < 112 so $4x(x+3) < 112$

$4x^2 + 12x - 112 < 0$

$x^2 + 3x - 28 < 0$

$(x+7)(x-4) < 0$

C.V.'s $x = 4$, $x = -7$

$-7 < x < 4$

but a negative length can't exist!

$\therefore \quad 0 < x < 4$

9:: Perimeter $= 2(4y + y + 3) = 10y + 6$

$20 < 10y + 6 < 54 \qquad (-6)$

$14 < 10y < 48 \qquad (\div 10)$

$1.4 < y < 4.8$

10i. $(x-5)^2 + (y+2)^2 = 25$

C at $(5, -2)$

radius $= \sqrt{25} = 5$ $\therefore$ diameter $= 10$

10ii. $(5, -2)$, $(7, 2)$

grad. $= \frac{-2-2}{5-7} = \frac{-4}{-2} = 2$

$y + 2 = 2(x-5)$

$y = 2x - 12$

10iii. $|CP| = \sqrt{(5-7)^2 + (-2-2)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$\sqrt{20} < 5$ $\therefore$ P is inside the circle

10iv. sub $y = 2x$ into equation of circle :

$(x-5)^2 + (2x+2)^2 = 25$

$x^2 - 10x + 25 + 4x^2 + 8x + 4 = 25$

$5x^2 - 2x + 4 = 0$

disc : $(-2)^2 - 4(5)(4) = -76$

disc < 0 $\therefore$ no real roots

$\therefore$ $y = 2x$ does not meet the circle.