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OCR

A Level

A Level Maths

OCR Core Maths C2 June 2011 Model Solutions

Name:

M M E

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Total Marks:

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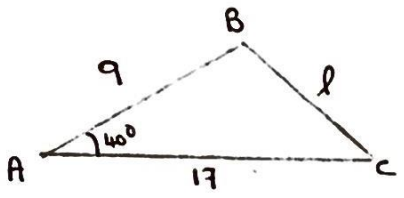
OCR June 11 C2

1i.



Cosine Rule : $l^2 = 9^2 + 17^2 - 2(9)(17)\cos 40°$

$l = 11.6$ (3 s.f.)

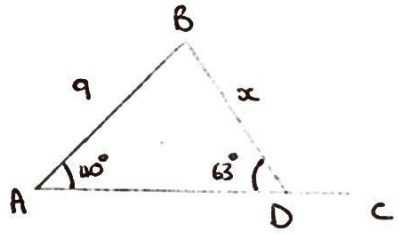
1ii.

Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(9)(17)\sin 40°$

$= 49.2$ (3 s.f.)

1iii.



Sine Rule : $\frac{x}{\sin 40°} = \frac{9}{\sin 63°}$

$x = \frac{9\sin 40°}{\sin 63°}$

$= 6.49$ (3 s.f.)

2i.

$\int 6x^{1/2} - 1 \; dx$

$= 4x^{3/2} - x + c$

2ii.

$\frac{dy}{dx} = 6x^{1/2} - 1$

$y = 4x^{3/2} - x + c$

when $x = 4$, $y = 17$ $\Rightarrow$ $17 = 4(4)^{3/2} - (4) + c$

$17 = 32 - 4 + c$

$c = -11$

$\therefore \quad y = 4x^{3/2} - x - 11$

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3i. Perimeter = Arc length + 2r (Arc length = rθ)

= 80 + 16

∴ 80 + 16 = 23.2

80 = 7.2

θ = 0.9c

3ii. Area = $\frac{1}{2}r^2\theta$

= $\frac{1}{2}(8)^2(0.69)$

= 28.8 (3 s.f.)

4i. $y = -1 + \sqrt{x+4}$

$y+1 = \sqrt{x+4}$

$(y+1)^2 = x+4$

$y^2+2y+1 = x+4$

$y^2+2y-3 = x$

4ii. $\int_1^3 y^2+2y-3\ dy$

$= \left[\frac{1}{3}y^3+y^2-3y\right]_1^3$

F[3] = 9

F[1] = -5/3

∴ $\int = 9-(-5/3)$

= $\frac{32}{3}$

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5i. $(3+kx)^5 = 3^5 + {}^5C_1(3)^4(kx) + {}^5C_2(3)^3(kx)^2 + {}^5C_3(3)^2(kx)^3 + \ldots$

$= 243 + 405kx + 270k^2x^2 + 90k^3x^3 + \ldots$

$a = 243$

5ii. $b = c \iff x$ coefficient $= x^2$ coefficient

$405k = 270k^2$

$405 = 270k$

$k = 3/2$

5iii. $d = 90(3/2)^3 = 303.75$

6i. $f(x) = x^3 + x^2 - 11x + 10$

$f(1) = (1)^3 + (1)^2 - 11(1) + 10 = 1$ ×

$f(-1) = (-1)^3 + (-1)^2 - 11(-1) + 10 = 21$ ×

$f(2) = (2)^3 + (2)^2 - 11(2) + 10 = 0$ ✓ $\Rightarrow (x-2)$ is a factor

6ii.

$$x-2 \;\overline{)\; x^3 + x^2 - 11x + 10}$$

$x^2 + 3x - 5$

$x^3 - 2x^2$ ↓

$3x^2 - 11x$

$3x^2 - 6x$

$-5x + 10$

$-5x + 10$

0

$\therefore f(x) = (x-2)(x^2 + 3x - 5)$

$x = 2$ or $x^2 + 3x - 5 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{29}}{2}$$

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7i. $a = 7, \quad r = -2$

$u_9 = ar^8 = 7(-2)^8 = 1792$

7ii. $S_{15} = \dfrac{7(1-(-2)^{15})}{1-(-2)} = 76,461$

7b. $a = 7, \quad d = -2, \quad S_N = -2900$

$S_N = \frac{1}{2}N(2a + (N-1)d) = -2900$

$\frac{1}{2}N(2(7) + (-2)(N-1)) = -2900$

$N(14 - 2N + 2) = -5800$

$14N - 2N^2 + 2N = -5800$

$2N^2 - 16N - 5800 = 0$

$N^2 - 8N - 2900 = 0$

$N = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(-2900)}}{2}$

$= 4 \pm 54$

$N = 58$ or -50,

N must be a positive integer

$\Rightarrow \quad N = 58$

8i. $y = 2^x \rightarrow y = 2^x - 3$ Translation -3 units in the positive y direction

8ii. when $x = 0$, $y = 2^0 - 3$

$= -2$

8iii. when $y = 0$, $2^x - 3 = 0$

$2^x = 3$

$x = \log_2 3$ (Since $\log_2(2^x) = x \log_2 2 = x$)

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8iv. when $x = p$, $y = 62$

$62 = 2^p - 3$

$2^p = 65$

$\log(2^p) = \log 65$

$p \log 2 = \log 65$

$p = \dfrac{\log 65}{\log 2}$

$= 6.02$ (3 s.f.)

8v.

x	y
3	5
3.5	$2^{3.5} - 3$
4	13

$h = 4 - 3/2 = 0.5$

Area $= \frac{1}{2} \cdot \frac{1}{2} \{ (5 + 13) + 2(2^{3.5} - 3) \}$

$= 8.66$ (3 s.f.)

9ai. $\cos x \rightarrow \cos 2x$ Stretch s.f. $1/2$ in x direction

Period of $\cos x = 2\pi$

$\therefore$ Period of $\cos 2x = 2\pi \times 1/2 = \pi$

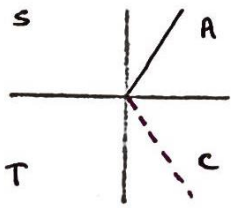
9aii. $(\pi/2, -1)$

9aiii. $\cos 2x \leq 0.5$ $\quad 0 \leq x \leq \pi$

$\cos \phi \leq 0.5$ $\quad$ let $\phi = 2x$

$0 \leq \phi \leq 2\pi$

P.V. $\phi = \pi/3$



$\phi = \pi/3, 5\pi/3$

$\therefore$ C.V.s $x = \pi/6, 5\pi/6$

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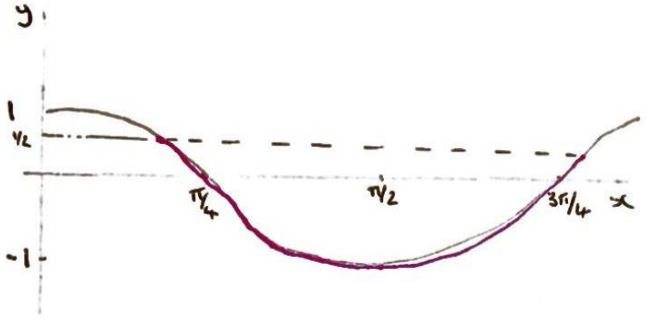
~~8cii.~~

9aii.



C.V. $x = \pi/6 , 5\pi/6$

$\therefore \ \pi/6 \leq x \leq 5\pi/6$

9b.

$\cos 2x = \sqrt{3} \sin 2x$ $\quad 0 \leq x \leq \pi$

$\cos\phi = \sqrt{3}\sin\phi \quad (\div \cos\phi)$ $\quad$ let $2x = \phi$

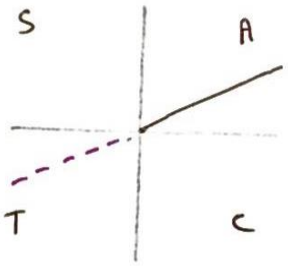
$0 \leq \phi \leq 2\pi$

use $\dfrac{\sin\theta}{\cos\theta} \equiv \tan\phi$

$1 = \sqrt{3}\tan\phi$

$\tan\phi = \dfrac{1}{\sqrt{3}}$

P.V. $\phi = \pi/6$



$\phi = \pi/6 , 7\pi/6$

$x = \dfrac{\pi}{12} , \dfrac{7\pi}{12}$

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