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OCR

A Level

A Level Maths

OCR Core Maths C1 June 2011

Model Solutions

Name:

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Total Marks:

OCR - Jan 11 C1

$3x^2 - 18x + 4$

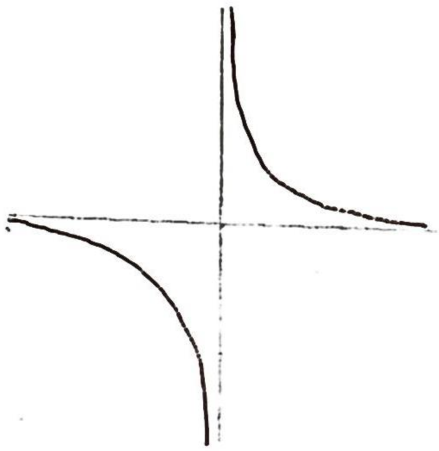
$3\left[x^2 - 6x\right] + 4$

$3\left[(x-3)^2 - 9\right] + 4$

$3(x-3)^2 - 27 + 4$

$3(x-3)^2 - 23$

i.



$y = \frac{1}{x}$

ii. $\frac{1}{x} \rightarrow \frac{1}{x} + 4$, translation 4 units in positive y direction

i. $\frac{(4x)^2 \times 2x^3}{x} = \frac{16x^2 \times 2x^3}{x} = \frac{32x^5}{x} = 32x^4$

ii. $(36x^{-2})^{-\frac{1}{2}}$

$= 36^{-\frac{1}{2}} . (x^{-2})^{-1/2}$

$= \frac{1}{\sqrt{36}} . x$

$= \frac{1}{6} x$

4. $y = 2(x-2)^2$ ①

$y + 3x = 26 \Rightarrow y = 26 - 3x$ ②

'equate ① and ②'

$2(x-2)^2 = 26 - 3x$

$2x^2 - 8x + 8 = 26 - 3x$

$2x^2 - 5x - 18 = 0$

$(2x - 9)(x + 2) = 0$

$x = 9/2$ or $x = -2$

$x = 9/2$; $y = 26 - 3\left(\frac{9}{2}\right)$

$= \frac{52}{2} - \frac{27}{2}$

$= \frac{25}{2}$

$x = -2$; $y = 26 - 3(-2)$

$= 26 + 6$

$= 32$

5i. $\sqrt{300} - \sqrt{48}$

$= \sqrt{100 \times 3} - \sqrt{16 \times 3}$

$= 10\sqrt{3} - 4\sqrt{3}$

$= 6\sqrt{3}$

5ii. $\frac{15 + \sqrt{40}}{\sqrt{5}} \times (\sqrt{5})$

$\frac{1}{5} \cdot (15\sqrt{5} + \sqrt{200})$

$= \frac{1}{5}(15\sqrt{5} + \sqrt{100 \times 2})$

$= \frac{1}{5}(15\sqrt{5} + 10\sqrt{2})$

$= 3\sqrt{5} + 2\sqrt{2}$

6. $3x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 4 = 0$

let $y = x^{1/4}$

$y^2 = x^{1/2}$

$3y^2 - 8y + 4 = 0$

$(3y-2)(y-2) = 0$

$y = 2/3$ or $y = 2$

$y = 2/3 \Rightarrow x^{1/4} = \frac{2}{3}$

$x = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

$y = 2 \Rightarrow x^{1/4} = 2$

$x = 2^4$

$= 16$

7i. $-9 \leq 6x + 5 \leq 0$ (-5)

$-14 \leq 6x \leq -5$ $(\div 6)$

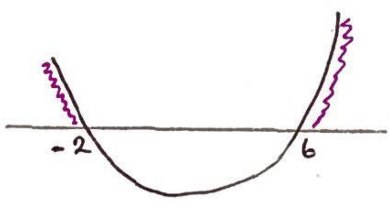
$-\frac{7}{3} \leq x \leq -\frac{5}{6}$

7ii. $6x + 5 < x^2 + 2x - 7$

$0 < x^2 - 4x - 12$

$(x-6)(x+2) > 0$

c.v. $x = 6$ $x = -2$



$x < -2$

$x > 6$

8i.

$y = 3x^2 - \frac{6}{x} - 2$

$y = 3x^2 - 6x^{-1} - 2$

$\frac{dy}{dx} = 6x + 6x^{-2}$

at stat. pt $\frac{dy}{dx} = 0$

$\therefore \quad 6x + \frac{6}{x^2} = 0 \quad (\times x^2)$

$6x^3 + 6 = 0$

$x^3 = -1$

$x = -1$

when $x = -1$, $y = 3(-1)^2 - \frac{6}{(-1)} - 2$

$= 3 + 6 - 2$

$= 7$

$\therefore$ stat pt. $(-1, 7)$

8ii.

$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$

at $(-1, 7)$ $\frac{d^2y}{dx^2} = 6 - \frac{12}{(-1)^3}$

$= 18$

$\frac{d^2y}{dx^2} > 0 \quad \therefore$ minimum point

9i.

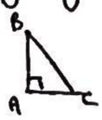
A $(1,3)$ B $(7,1)$ C $(-3,-9)$

length AB $= \sqrt{(7-1)^2 + (1-3)^2} = \sqrt{40}$

AC $= \sqrt{(1--3)^2 + (3--9)^2} = \sqrt{160}$

BC $= \sqrt{(7--3)^2 + (1--9)^2} = \sqrt{200}$

$BC^2 = AC^2 + AB^2 \quad \therefore$ Pythagoras' Theorem holds

$\Rightarrow$ 

9ii. A, B and C lie on the circumference

$\therefore$ BC is a diameter

radius $= \frac{1}{2}BC = \frac{\sqrt{200}}{2} = 5\sqrt{2}$ (prev. part)

Midpoint of BC must be the centre of circle

$\left(\frac{7+-3}{2}, \frac{1+-9}{2}\right) = (2, -4)$

$\therefore \quad (x-2)^2 + (y+4)^2 = (5\sqrt{2})^2$

$(x-2)^2 + (y+4)^2 = 50$

$x^2 - 4x + 4 + y^2 + 8y + 16 = 50$

$x^2 + y^2 - 4x + 8y - 30 = 0$

10i. $y = (2x-1)(x+3)(x-1)$

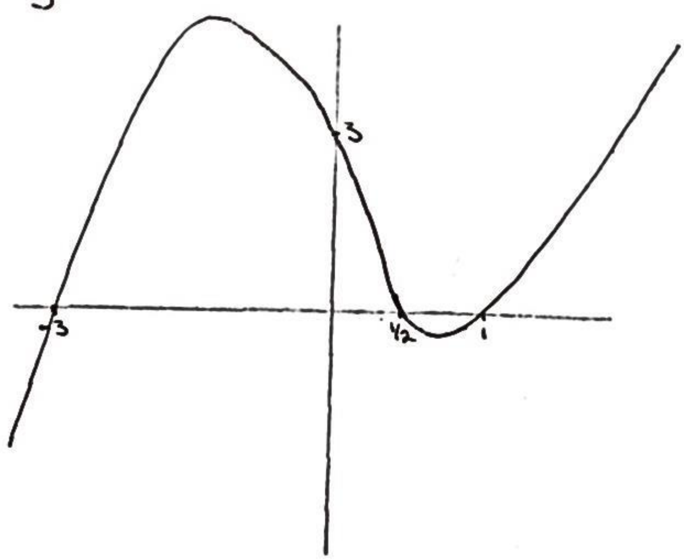
roots at $x = -3, 1, 1/2$

crosses y axis when $x = 0$

$y = (2(0)-1)(0+3)(0-1)$

$= (-1)(3)(-1)$

$= 3$



10ii.

$$y = (2x-1)(x+3)(x-1)$$
$$= (2x^2+5x-3)(x-1)$$
$$= 2x^3 - 2x^2 + 5x^2 - 5x - 3x + 3$$
$$= 2x^3 + 3x^2 - 8x + 3$$

$$\frac{dy}{dx} = 6x^2 + 6x - 8$$

at P, $\frac{dy}{dx} = 6(1)^2 + 6(1) - 8$
$$= 6 + 6 - 8$$
$$= 4$$

10iii. l is parallel $\therefore$ grad of $l = 4$

when $x = -2$, $y = 2(-2)^3 + 3(-2)^2 - 8(-2) + 3$
$$= -16 + 12 + 16 + 3$$
$$= 15$$

$$y - 15 = 4(x+2)$$
$$y = 4x + 23$$

10iv. When $x = -2$, $\frac{dy}{dx} = 6(-2)^2 + 6(-2) - 8$
$$= 24 - 12 - 8$$
$$= 4$$

Grad of ~~[illegible]~~ line = grad of curve at $x = -2$

$\therefore$ the line is a tangent.