OCR

A Level

A Level Maths

OCR Core Maths C2 June 2010

Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

OCR June 10 C2

1i: $f(x) = x^3 + ax^2 - ax - 14$

$(x-2)$ is a factor $\therefore f(2) = 0$

$f(2) = (2)^3 + a(2)^2 - a(2) - 14$

$0 = 8 + 4a - 2a - 14$

$2a = 6$

$a = 3$

1ii. $f(x) = x^3 + 3x^2 - 3x - 14$

$f(-1) = (-1)^3 + 3(-1)^2 - 3(-1) - 14$

$= -9$

2i. $\int_1^{10} \sqrt[3]{7+x}\, dx \qquad h = \frac{10-1}{3} = 3$

x	y
1	2
4	$\sqrt[3]{11}$
7	$\sqrt[3]{14}$
10	$\sqrt[3]{17}$

$\int \approx \frac{3}{2}\left\{(2+\sqrt[3]{17}) + 2(\sqrt[3]{11} + \sqrt[3]{14})\right\}$

$= 20.7592\ldots$

$= 20.8 \quad (3\text{ s.f.})$

2ii. Use more strips to obtain a better estimate

3i. $(1+\frac{1}{2}x)^{10} = 1^{10} + {}^{10}C_1 1^9 (\frac{1}{2}x) + {}^{10}C_2 1^8 (\frac{1}{2}x)^2 + {}^{10}C_3 1^7 (\frac{1}{2}x)^3 + \ldots$

$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

3ii.

$$(3 + 4x + 2x^2)(1 + \tfrac{1}{2}x)^{10}$$

$$(3 + 4x + 2x^2)\left(1 + 5x + \frac{45}{4}x^2 + 15x^3\right)$$

Only want x^3 :

$$3 \times 15x^3 = 45x^3$$

$$4x \times \frac{45}{4}x^2 = 45x^3$$

$$2x^2 \times 5x = 10x^3$$

$\therefore \quad 100x^3$ coefficient is 100

4i.

$$u_n = 5n + 1$$

$$u_1 = 5(1) + 1 = 6$$

$$u_2 = 5(2) + 1 = 11$$

$$u_3 = 5(3) + 1 = 16$$

4ii.

$\sum_{n=1}^{40} u_n$ Arithmetic $\Rightarrow$ $a = 6$ $d = 5$

$$S_{40} = \frac{40}{2}\left\{2(6) + (40-1)5\right\} = 4140$$

4iii.

$w_1 = 2$ $\quad w_{n+1} = 5w_n + 1$

$$w_2 = 5(2) + 1 = 11$$

$$w_3 = 5(11) + 1 = 56$$

$$\therefore \quad u_p = 56$$

$$56 = a + (n-1)d$$

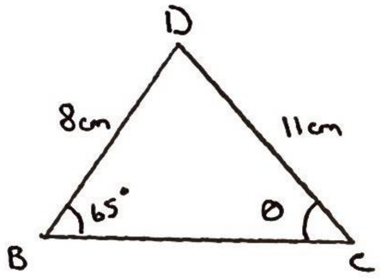
$$= 6 + (p-1)5$$

$$50 = 5(p-1)$$

$$10 = p - 1$$

$$\therefore \quad p = 11$$

5i.



$$\frac{\sin\theta}{8} = \frac{\sin 65}{11}$$

$$\sin\theta = \frac{8\sin 65}{11}$$

$$\theta = 41.2^\circ \quad (3\text{ s.f.})$$

5ii. Triangles are congruent so $A\hat{B}E = C\hat{B}D = 65^\circ$

$$\therefore \quad E\hat{B}D = 180 - 65^\circ - 65^\circ$$
$$= 50^\circ$$

To convert to radians multiply by $\frac{\pi}{180}$

$$50 \times \frac{\pi}{180} = 0.873 \quad (3\text{ s.f.})$$

5b. Area of sector $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(8)^2(0.873)$$
$$= 27.9253$$

Area of Δ BED $= \frac{1}{2}ab\sin C$

$$= \frac{1}{2}(8)(8)\sin 0.873$$
$$= 24.5203$$

Shaded area $=$ sector $- \Delta$

$$= 3.41 \quad (3\text{ s.f.})$$

6a. $\int_3^5 x^2 + 4x \, dx$

$$= \left[\frac{1}{3}x^3 + 2x^2\right]_3^5$$

$F[5] = 275/3$

$F[3] = 27 \qquad \therefore \int = \frac{275}{3} - 27 = \frac{194}{3}$

6b. $\int 2-6\sqrt{y} \quad dy$

$= \int 2-6y^{1/2} \quad dy$

$= 2y - 4y^{3/2} + c$

6c. $\int_1^\infty \frac{8}{x^3} \, dx = \int_1^a 8x^{-3} \, dx$

$= \left[-4x^{-2}\right]_1^a$

$F[a] = -\frac{4}{a^2}$

$F[1] = -4$

as $a \to \infty \quad \frac{-4}{a^2} \to 0$

$\therefore \int = 0-(-4) = 4$

7i. $\frac{\sin^2 x - \cos^2 x}{1-\sin^2 x} \equiv \tan^2 x - 1$

LHS: $\frac{\sin^2 x - \cos^2 x}{1-\sin^2 x}$

USE $\sin^2 x + \cos^2 x \equiv 1 \quad \forall x \in \mathbb{R}$

$\therefore \cos^2 x \equiv 1 - \sin^2 x$

$\frac{\sin^2 x - \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

$= \tan^2 x - 1 = \text{RHS}$

7ii. $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x$ $\qquad 0^\circ \leq x \leq 360^\circ$

$\tan^2 x - 1 = 5 - \tan x$

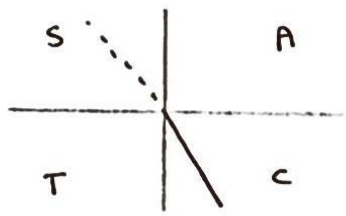
$\tan^2 x + \tan x - 6 = 0$

$(\tan x + 3)(\tan x - 2) = 0$

$\tan x = -3$ or $\tan x = 2$

$\tan x = -3$

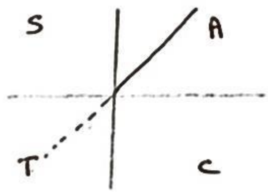
P.V. -71.565



$x = 108^\circ, 288^\circ$

$\tan x = 2$

P.V. 63.4°



$x = 63.4^\circ, 243^\circ$ (3 s.f.)

8a. $5^{3w-1} = 4^{250}$

$\log(5^{3w-1}) = \log(4^{250})$

$(3w-1)\log 5 = 250 \log 4$

$3w \log 5 - \log 5 = 250 \log 4$

$3w \log 5 = 250 \log 4 + \log 5$

$w = \dfrac{250 \log 4 + \log 5}{3 \log 5}$

$= 72.1$ (3sf)

8b. $\log_x(5y+1) - \log_x 3 = 4$

$\log_x\left(\dfrac{5y+1}{3}\right) = 4$

$x^4 = \dfrac{5y+1}{3}$

$3x^4 = 5y+1 \implies y = \dfrac{3x^4 - 1}{5}$

9i. G.P. A.P.

G.P.	A.P.
$u_1 : a$	$u_1 : a$
$u_2 : a^2 r$	$u_2 : ar$
$u_3 : ar^2$	$u_3 : ar^3$
$u_4 : ar^3$	

In an A.P. $u_3 - u_2 = u_2 - u_1$

$$ar^3 - ar = ar - a \quad (\div a)$$

$$r^3 - r = r - 1$$

$$r^3 - 2r + 1 = 0$$

9ii. G.P. converges $\therefore$ $|r| < 1 \iff -1 < r < 1$

let $f(r) = r^3 - 2r + 1$

$f(1) : 1^3 - 2(1) + 1 = 0 \quad \therefore (r-1)$ is a factor

$$\begin{array}{r|l} & r^2 + r - 1 \\ \hline r - 1 & r^3 + 0r^2 - 2r + 1 \\ & r^3 - r^2 \\ \hline & r^2 - 2r \\ & r^2 - r \\ \hline & -r + 1 \\ & -r + 1 \\ \hline & 0 \end{array}$$

$\therefore f(r) = (r-1)(r^2 + r - 1)$

$r = 1$ or $r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

r must be between 1 and -1

$$\therefore r = \frac{-1 + \sqrt{5}}{2}$$

9iii. $$3 + \sqrt{5} = \frac{a}{1 - \left(\frac{-1+\sqrt{5}}{2}\right)}$$

$$a = (3 + \sqrt{5})\left(1 - \left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$= 2$$