OCR

A Level

A Level Maths

OCR Core Maths C2 January 2013 Model Solutions

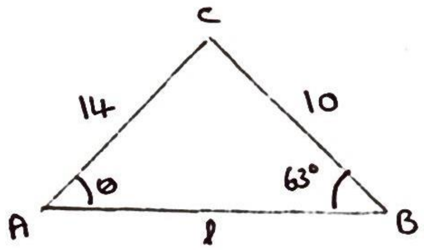
Name:

Mathsmadeeasy.co.uk

Total Marks:

OCR Jan 13 C2

1i.



Sine Rule, $\frac{\sin\theta}{10} = \frac{\sin 63^\circ}{14}$

$\sin\theta = \frac{10\sin 63}{14}$

$\theta = 39.5^\circ$ (3 s.f.)

1ii. $A\hat{C}B = 180 - 63 - 39.5 = 77.47...$

Cosine Rule: $l^2 = 14^2 + 10^2 - 2(14)(10)\cos(77.47...)$

$l = 15.3$ (3 s.f.)

2i. $u_1 = 7$, $u_{n+1} = u_n + 4$

Arithmetic Progression: $a = 7$, $d = 4$

$u_{17} = a + (17-1)d$

$= 7 + 16(4)$

$= 7 + 64$

$= 71$

2ii. $\sum_{n=1}^{35} u_n = \sum_{n=36}^{50} u_n$

$S_{35} = \frac{35}{2}\left(2(7) + 34(4)\right) = 2625$

$\sum_{n=36}^{50} u_n = S_{50} - S_{35}$

$S_{50} = \frac{50}{2}\left(2(7) + 49(4)\right) = 5250$

$\therefore \sum_{n=36}^{50} u_n = 5250 - 2625$

$= 2625$

$= \sum_{n=1}^{35} u_n$

3i. $\frac{dy}{dx} = kx(2x-1)$ $\quad P(2,7)$

at $(2,7)$, $\frac{dy}{dx} = 9 \Rightarrow 9 = 2k(2(2)-1)$

$9 = 6k$

$k = 3/2$

3ii. $y = \int \frac{3}{2}x(2x-1)\ dx$

$= \int 3x^2 - \frac{3}{2}x\ dx$

$= x^3 - \frac{3}{4}x^2 + c$

when $y = 7,\ x = 2$: $7 = (2)^3 - \frac{3}{4}(2)^2 + c$

$7 = 8 - 3 + c$

$c = 2$

$y = x^3 - \frac{3}{4}x^2 + 2$

4i. $(2+x)^5 = 2^5 + {}^5C_1 2^4 x + {}^5C_2 2^3 x^2 + {}^5C_3 2^2 x^3 + {}^5C_4 2x^4 + {}^5C_5 x^5$

$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$

4ii. $(2+3y+y^2)^5$ i.e. $x = (3y+y^2)$

we want the y^3 terms : $x^2 = (3y+y^2)^2$

$= 9y^2 + 6y^3 + \dots$

$80x^2 \Rightarrow 80 \times 6y^3 = 480y^3$

$x^3 = (3y+y^2)^3 = (3y+y^2)(9y^2+6y^3+\dots)$

$= \dots + 27y^3 + \dots$

$40x^3 \Rightarrow 40 \times 27y^3 = 1080y^3$

$480y^3 + 1080y^3 = 1560y^3$

5i.

$$2\sin x = \frac{4\cos x - 1}{\tan x} \qquad (\times \tan x)$$

$$2\sin x . \frac{\sin x}{\cos x} = 4\cos x - 1 \qquad (\times \cos x)$$

$$2\sin^2 x = 4\cos^2 x - \cos x \qquad (\text{use } \sin^2 x \equiv 1 - \cos^2 x)$$

$$2(1 - \cos^2 x) = 4\cos^2 x - \cos x$$

$$2 - 2\cos^2 x = 4\cos^2 x - \cos x$$

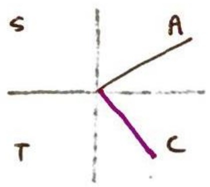
$$6\cos^2 x - \cos x - 2 = 0$$

5ii.

$$(3\cos x - 2)(2\cos x + 1) = 0 \qquad 0 \leq x \leq 360°$$

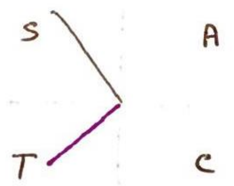
$\cos x = 2/3$ or $\cos x = -1/2$

P.V. $x = 48.2°$



$x = 48.2°, \ 311.8°$

P.V. $x = 120°$



$x = 120°, \ 240°$

6i.

$u_1 = 2x$
$u_2 = x + 4$
$u_3 = 2x - 7$

AP $\therefore$

$$u_2 - u_1 = u_3 - u_2$$

$$x + 4 - 2x = 2x - 7 - (x + 4)$$

$$4 - x = x - 11$$

$$15 = 2x$$

$$x = 7.5$$

6iia.

$x = 8 \quad \therefore \quad u_1 = 16$
$u_2 = 12$
$u_3 = 9$

G.P. if

$$\frac{u_2}{u_1} = \frac{u_3}{u_2}$$

$$\frac{12}{16} = \frac{9}{12}$$

$$\frac{3}{4} = \frac{3}{4} = r \quad \therefore \text{ G.P.}$$

$$S_\infty = \frac{a}{1-r} = \frac{16}{1 - 3/4} = 64$$

6iib. GP $\Rightarrow \dfrac{u_2}{u_1} = \dfrac{u_3}{u_2}$

$$\frac{x+4}{2x} = \frac{2x-7}{x+4}$$

$$(x+4)^2 = 2x(2x-7)$$

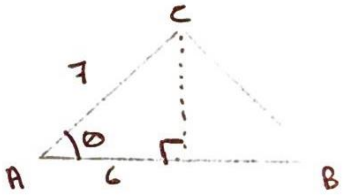
$$x^2+8x+16 = 4x^2-14x$$

$$3x^2-22x-16=0$$

$$(x-8)(3x+2)=0$$

$$x=8 \quad \text{or} \quad x=-2/3$$

7i.



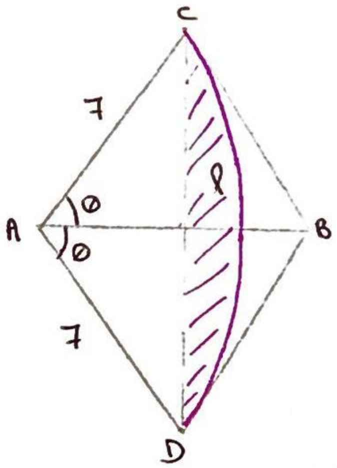
$$\cos\theta = \frac{6}{7}$$

$$\theta = \cos^{-1}\left(\frac{6}{7}\right)$$

$$= 0.541099\ldots$$

$$= 0.5411 \quad (4\text{ s.f.})$$

7ii.



$$l = r\theta$$

$$= 7(2\theta) \quad (\text{for pink line})$$

$$= 7.57539\ldots$$

Perimeter of shaded region $= 2 \times$ pink line

$$= 15.2 \quad (3\text{s.f.})$$

7iii. $\frac{1}{2}$ Shaded area $=$ Area of Sector ACD $-$ Area of Δ ADC

Area of Sector : $\frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}(7)^2.(0.541099\ldots \times 2)$

$$= 26.5138\ldots$$

Area of Δ : $\frac{1}{2}ab\sin C \Rightarrow \frac{1}{2}(7)(7)\sin(0.541099\ldots \times 2)$

$$= 21.633\ldots$$

7iii. $\frac{1}{2}$ Shaded Area $= 4.880\ldots$

$\therefore$ Total Shaded Area $= 9.76$ (3 s.f.)

8i. $\log_2 x \rightarrow \log_2(x-3)$ Translation 3 units in positive x direction

8ii. when $y = 3$, $x = a$ $\therefore$ $3 = \log_2 a$

$a = 2^3$

$= 8$

8iii. when $y = 1.8$, $x = b$ $\therefore$ $1.8 = \log_2(b-3)$

$b - 3 = 2^{1.8}$

$b = 3 + 2^{1.8}$

$= 6.48$ (3 s.f.)

8iv. Difference in y coordinates $= 4$

$\therefore \log_2 x - \log_2(x-3) = 4$

$\log_2\left(\frac{x}{x-3}\right) = 4$

$\frac{x}{x-3} = 2^4$

$x = 16(x-3)$

$x = 16x - 48$

$15x = 48$

$x = 16/5$

9i.

$$\int_a^{2a} \frac{2x^3 - 5x^2 + 4}{x^2} \, dx = 0$$

$$\int_a^{2a} 2x - 5 + 4x^{-2} \, dx$$

$$\left[x^2 - 5x - \frac{4}{x} \right]_a^{2a} = 0$$

$$(2a)^2 - 5(2a) - \frac{4}{2a} - \left(a^2 - 5a - \frac{4}{a} \right) = 0$$

$$4a^2 - 10a - \frac{4}{2a} - a^2 + 5a + \frac{4}{a} = 0 \qquad (\times a)$$

$$3a^3 - 5a^2 - \frac{4}{2} + 4 = 0$$

$$3a^3 - 5a^2 + 2 = 0$$

9ii.

$$F(1) = 3(1)^3 - 5(1)^2 + 2$$

$$= 3 - 5 + 2 = 0 \quad \therefore \ a = 1 \text{ is a root}$$

$$\begin{array}{r l}
 & 3a^2 - 2a - 2 \\
a-1 \,\big) & 3a^3 - 5a^2 + 0a + 2 \\
 & \underline{3a^3 - 3a^2} \\
 & -2a^2 + 0a \\
 & \underline{-2a^2 + 2a} \\
 & -2a + 2 \\
 & \underline{-2a + 2} \\
 & 0
\end{array}$$

$$\therefore \ (a-1)(3a^2 - 2a - 2)$$

if $3a^2 - 2a - 2 = 0$

$$a = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-2)}}{6}$$

$$= \frac{1 \pm \sqrt{7}}{3}$$

$$a > 0 \quad \therefore \ a = 1 \text{ or } \frac{1 + \sqrt{7}}{3}$$