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OCR

A Level

A Level Maths

OCR Core Maths C1 January 2013 Model Solutions

Name:

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Total Marks:

OCR - Jan 13 C1

1i. $x^2 - 6x - 2 = 0$

$(x-3)^2 - 9 - 2 = 0$

$(x-3)^2 = 11$

$x - 3 = \pm\sqrt{11}$

$x = 3 \pm \sqrt{11}$

1ii. $y = x^2 - 6x - 2$

$\frac{dy}{dx} = 2x - 6$

at $x = -5$, $\frac{dy}{dx} = 2(-5) - 6$

$= -10 - 6$

$= -16$

2i. $3^n = 1$ $\quad n = 0$

2ii. $t^{-3} = 64$

$\frac{1}{t^3} = 64$

$t^3 = 1/64$

$t = 1/4$

2iii. $(8p^6)^{\frac{1}{3}} = 8$

$8^{1/3} \cdot (p^6)^{1/3} = 8$

$2p^2 = 8$

$p^2 = 4$

$p = \pm 2$

3i.

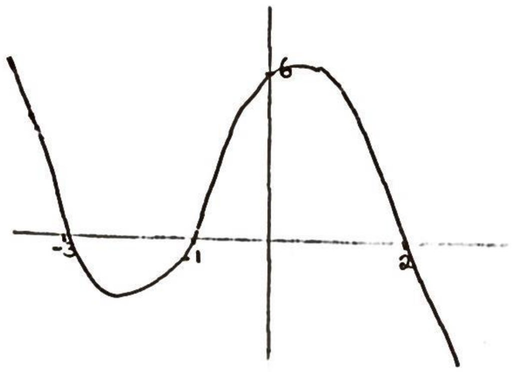
$y = (1+x)(2-x)(3+x)$

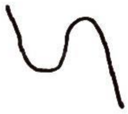
roots at $x = -1, 2, -3$

when $x = 0$, $y = 1 \times 2 \times 3$

$= 6$

so crosses y axis at 6



negative x^3 so 

3ii.

$(1+x)(2-x)(3+x) \longrightarrow (1-x)(2+x)(3-x)$ reflection in y axis

Since x replaced by $-x$

4i.

$y = 2x^2 - 3x - 5 \Rightarrow 2y = 4x^2 - 6x - 10$ ①

$10x + 2y + 11 = 0 \Rightarrow 2y = -10x - 11$ ②

'Equate ① and ②'

$4x^2 - 6x - 10 = -10x - 11$

$4x^2 + 4x + 1 = 0$

$(2x+1)^2 = 0$

$x = -1/2$

$x = -1/2$, $y = 2(-\frac{1}{2})^2 - 3(-1/2) - 5$

$y = \frac{1}{2} + \frac{3}{2} - 5$

$= -3$

$\therefore$ $x = -1/2$, $y = -3$

4ii.

Only 1 solution, i.e. 1 point of intersection

$\therefore$ line is a tangent.

5i. $(x+4)(5x-3) - 3(x-2)^2$

$5x^2 + 17x - 12 - 3(x^2 - 4x + 4)$

$5x^2 + 17x - 12 - 3x^2 + 12x - 12$

$2x^2 + 29x - 24$

5ii. $(x+3)(x+k)(2x-5)$

$(x^2 + 3x + kx + 3k)(2x - 5)$

x^2's : $-5x^2 + 6x^2 + 2kx^2$

so $-5 + 6 + 2k = -3$

$2k = -4$

$k = -2$

6i. $(-2, 7)$ $(-4, p)$

grad $= 4$ $\therefore$ $\dfrac{p-7}{-4--2} = 4$

$\dfrac{p-7}{-2} = 4$

$p - 7 = -8$

$p = -1$

6ii. Mid of $(-2, 7)$ $(6, q)$ $= (m, 5)$

$\left(\dfrac{-2+6}{2}, \dfrac{7+q}{2}\right)$

x's $\dfrac{-2+6}{2} = m \Rightarrow 4 = 2m, \quad m = 2$

y's $\dfrac{7+q}{2} = 5 \Rightarrow 7 + q = 10, \quad q = 3$

6iii. $(-2, 7)$ $(d, 3)$ Length $= 2\sqrt{13}$

$\therefore$ $2\sqrt{13} = \sqrt{(d--2)^2 + (3-7)^2}$

$2\sqrt{13} = \sqrt{(d+2)^2 + (-4)^2}$ (square both sides)

$52 = (d+2)^2 + 16$

$d^2 + 4d + 4 + 16 = 52$

$d^2 + 4d - 32 = 0$

$(d+8)(d-4) = 0$

$d = 4$ or $d = -8$

7i. $y = \dfrac{(3x)^2 \times x^4}{x}$

$= \dfrac{9x^6}{x}$

$= 9x^5$

$\dfrac{dy}{dx} = 45x^4$

7ii. $y = \sqrt[3]{x}$

$= x^{1/3}$

$\dfrac{dy}{dx} = \dfrac{1}{3}x^{-2/3}$

7iii. $y = \dfrac{1}{2x^3}$

$= \dfrac{1}{2}x^{-3}$

$\dfrac{dy}{dx} = -\dfrac{3}{2}x^{-4}$

8. $kx^2 + (3k-1)x - 4 = 0$

No real roots $\therefore$ disc < 0

$$(3k-1)^2 - 4(k)(-4) < 0$$

$$9k^2 - 6k + 1 + 16k < 0$$

$$9k^2 + 10k + 1 < 0$$

$$(9k+1)(k+1) < 0$$

C.V.s $k = -1$ $k = -1/9$

$$-1 < k < -\frac{1}{9}$$

9i. $x^2 + y^2 - 2x + 10y - 19 = 0$

$$(x-1)^2 - 1 + (y+5)^2 - 25 - 19 = 0$$

$$(x-1)^2 + (y+5)^2 = 45$$

Centre at $(1, -5)$

Radius $= \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

9ii. sub $x = 7$, $y = -2$ in circle

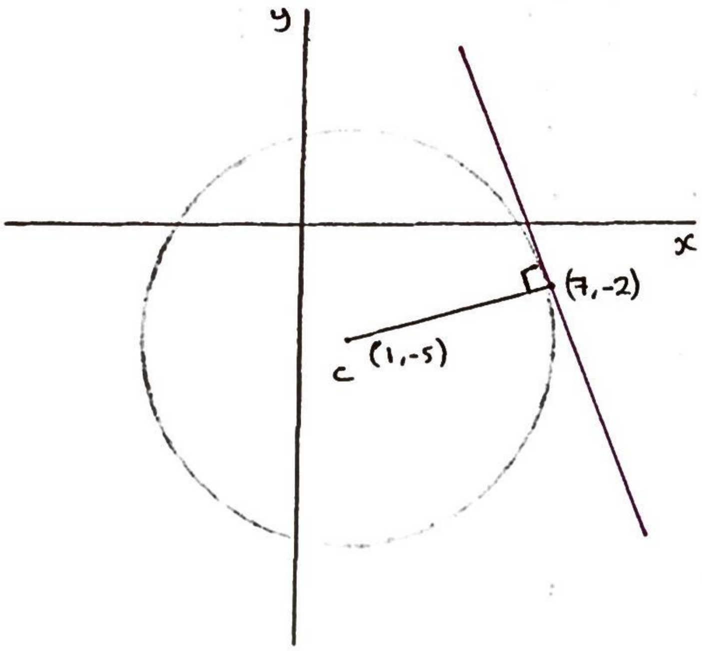
$$(7-1)^2 + (-2+5)^2 = 45$$

$$36 + 9 = 45$$

$$45 = 45 \checkmark$$

$\therefore$ point lies on circumference of circle

9iii.



grad of C to (7,-2) $= \dfrac{-2--5}{7-1} = \dfrac{1}{2}$

$\therefore$ grad of tangent $= -2$ (since $\perp$)

$y+2 = -2(x-7)$

$y+2 = -2x+14$

$y+2x-12 = 0$

10. $y = \frac{1}{3}x^3 + \frac{9}{x}$

$= \frac{1}{3}x^3 + 9x^{-1}$

$\frac{dy}{dx} = x^2 - \frac{9}{x^2}$

grad of '$y = 8x+3$' $= 8$

$\therefore \frac{dy}{dx} = 8$; $8 = x^2 - \frac{9}{x^2}$ $(\times x^2)$

$8x^2 = x^4 - 9$

$x^4 - 8x^2 - 9 = 0$

10 cont.

$x^4 - 8x^2 - 9 = 0$ let $y = x^2$

$y^2 = x^4$

$y^2 - 8y - 9 = 0$

$(y-9)(y+1) = 0$

$y = 9$ or $y = -1$

$y = 9$; $x^2 = 9$

$x = \pm 3$

$y = -1$; $x^2 = -1$

$x = \sqrt{-1}$ ✗

$x = 3$; $y = \frac{1}{3}(3)^3 + \frac{9}{3}$

$= 9 + 3$

$= 12$

$x = -3$; $y = \frac{1}{3}(-3)^3 + \frac{9}{(-3)}$

$= -9 - 3$

$= -12$

$\therefore$ points at $(3, 12)$ $(-3, -12)$