OCR

A Level

A Level Maths

OCR Core Maths C1 January 2012 Model Solutions

Name:

M M E

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Total Marks:

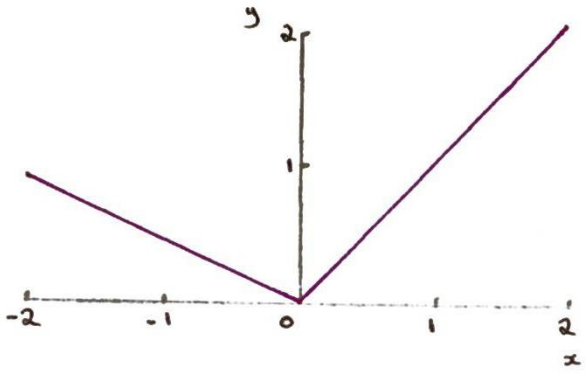
OCR - Jan 12 C1

1. $\dfrac{15+\sqrt{3}}{3-\sqrt{3}}$, $\dfrac{(15+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}$

$= \dfrac{45+18\sqrt{3}+3}{9-3}$

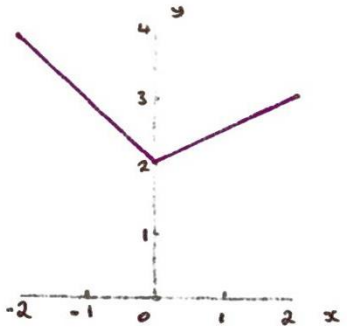
$= 8+3\sqrt{3}$

2i.



$f(x) \rightarrow f(-x)$ reflection in y axis

2ii.



$f(x) \rightarrow f(x)+2$ translation 2 units in positive y direction

3. $5x^2+px-8 = q(x-1)^2+r$

$q=5$ (comparing coefficients of x^2)

$5x^2+px-8 = 5(x-1)^2+r$

$= 5(x^2-2x+1)+r$

$= 5x^2-10x+r+5$

$\therefore$ $p=-10$ (x coefficients)

$r+5=-8$ (constants)

$r=-13$

4i. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

4ii. $16^{3/4} = \left(16^{\frac{1}{4}}\right)^3 = 2^3 = 8$

4iii. $\frac{\sqrt{200}}{\sqrt{8}} = \frac{\sqrt{100 \times 2}}{\sqrt{4 \times 2}} = \frac{10\sqrt{2}}{2\sqrt{2}} = \frac{10}{2} = 5$

5. $\frac{3}{y^4} - \frac{10}{y^2} - 8 = 0 \qquad \times y^4$

$3 - 10y^2 - 8y^4 = 0$

$8y^4 + 10y^2 - 3 = 0$ let $x = y^2$, $x^2 = y^4$

$8x^2 + 10x - 3 = 0$

$(4x - 1)(2x + 3) = 0$

$x = \frac{1}{4}$ or $x = -3/2$

$y^2 = \frac{1}{4}$ $\qquad y^2 = -3/2$

$y = \pm \frac{1}{2}$ $\qquad y = \sqrt{-3/2}$ ×

6i. $f(x) = \frac{4}{x} - 3x + 2$

$= 4x^{-1} - 3x + 2$

$f'(x) = -4x^{-2} - 3$

6ii. $f''(x) = 8x^{-3}$

$f''(\frac{1}{2}) = \frac{8}{(1/2)^3}$

$= \frac{8}{1/8}$

$= 64$

7i. $y = (x+2)(x^2 - 3x + 5)$

$= x^3 - 3x^2 + 5x + 2x^2 - 6x + 10$

$= x^3 - x^2 - x + 10$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

At stat. point $\frac{dy}{dx} = 0$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -1/3$ or $x = 1$

$\frac{d^2y}{dx^2} = 6x - 2$

at $x = -1/3$, $\frac{d^2y}{dx^2} = 6(-\frac{1}{3}) - 2 = -4 < 0 \Rightarrow$ max pt.

at $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2 = 4 > 0 \Rightarrow$ min pt.

when $x = 1$, $y = (1)^3 - (1)^2 - (1) + 10$

$= 9$

$\therefore$ minimum at $(1, 9)$

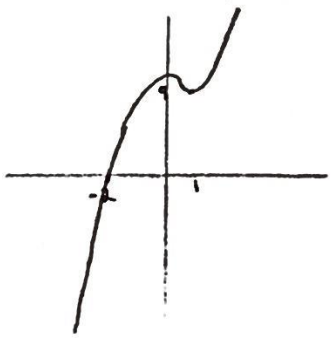
7ii. $x^2 - 3x + 5$

disc : $(-3)^2 - 4(1)(5)$

$= 9 - 20$

$= -11$

7iii. Disc. of quadratic term < 0, $\therefore$ only crosses the x axis once at -2.



8. Equation of l: $y-5 = -2(x-3)$

$y-5 = -2x+6$

$y = 11-2x$ ①

To have a length of $6\sqrt{5}$:

$(x-3)^2 + (y-5)^2 = (6\sqrt{5})^2$ ②

'Sub ① into ②'

$(x-3)^2 + (11-2x-5)^2 = 180$

$(x-3)^2 + (6-2x)^2 = 180$

$x^2 - 6x + 9 + 36 - 24x + 4x^2 = 180$

$5x^2 - 30x - 135 = 0$

$x^2 - 6x - 27 = 0$

$(x-9)(x+3) = 0$

$x = 9$ or $x = -3$

$x=9$; $y = 11-2(9)$

$= 11-18$

$= -7$

$x=-3$; $y = 11-2(-3)$

$= 11+6$

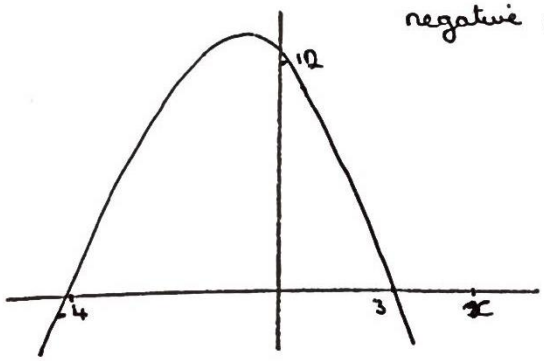
$= 17$

9. $y = 12-x-x^2$

$= -(x-3)(x+4)$ so roots at $x=3$, $x=-4$

when $x=0$, $y=12$

negative x^2 so $\cap$ shaped

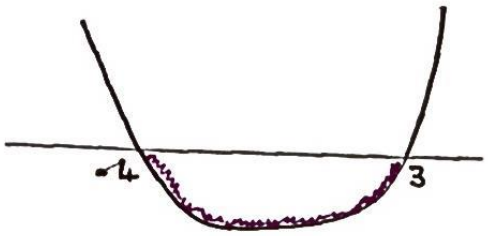


7iii. $12 - x - x^2 > 0$

$x^2 + x - 12 < 0$

$(x + 4)(x - 3) < 0$

c.v.s $x = -4$, $x = 3$



$-4 < x < 3$

9iii. $y = 12 - x - x^2$ ①

$3x + y = 4 \Rightarrow y = 4 - 3x$ ②

'① = ②'

$12 - x - x^2 = 4 - 3x$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4$ or $x = -2$

$x = 4$; $y = 4 - 3(4)$

$= 4 - 12$

$= -8$

$x = -2$; $y = 4 - 3(-2)$

$= 4 + 6$

$= 10$

So points of intersection are $(4, -8)$, $(-2, 10)$

10i. $(x+2)^2 + (y-4)^2 = 5^2$

$x^2 + 4x + 4 + y^2 - 8y + 16 - 25 = 0$

$x^2 + y^2 + 4x - 8y - 5 = 0$

10ii. grad of CP $= \frac{8-4}{-5+2} = -\frac{4}{3}$ P $(-5, 8)$

$\therefore$ grad of tangent $= 3/4$

$y - 8 = \frac{3}{4}(x+5)$ $(\times 4)$

$4y - 32 = 3x + 15$

$3x - 4y + 47 = 0$

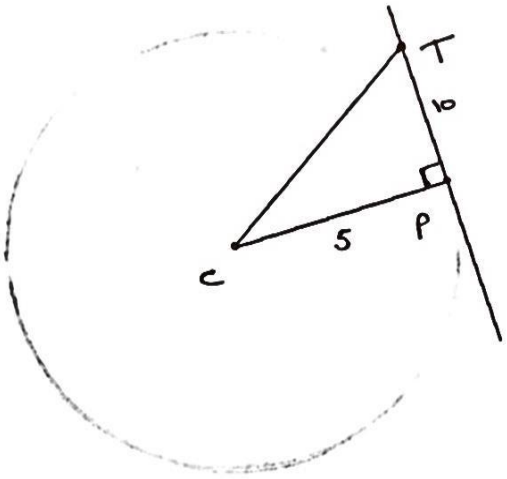
10iii. at T, $x = 3$ $y = 14$

$3(3) - 4(14) + 47 = 0$

$9 - 56 + 47 = 0$

$-47 + 47 = 0$ ✓ $\therefore$ T lies on the tangent

10iv.



Length TP : $\sqrt{(3--5)^2 + (14-8)^2}$

$= \sqrt{64+36}$

$= 10$

$\therefore$ Area of $\Delta = \frac{1}{2}(5 \times 10)$

$= 25$