OCR

A Level

A Level Maths

OCR Core Maths C2 January 2011 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

OCR - Jan 11 C2

1i. $(1+2x)^7 = 1^7 + {}^7C_1 . 1^6(2x) + {}^7C_2 . 1^5(2x)^2$

$= 1 + 14x + 84x^2 + \ldots$

1ii. $(2-5x)(1+14x+84x^2+\ldots)$

x^2 terms : $2(84x^2) - 5x(14x)$

$= 98x^2$

2i. $u_n = 3n+2$

$u_1 = 3(1)+2 = 5$

$u_2 = 3(2)+2 = 8$

$u_3 = 3(3)+2 = 11$

2ii. Common difference of 3 $\therefore$ arithmetic sequence $(a=5,\ d=3)$

2iii. $\sum_{n=101}^{200} u_n = \sum_{n=1}^{200} u_n - \sum_{n=1}^{100} u_n$

$= S_{200} - S_{100}$

$= \frac{200}{2}\left[2(5)+(200-1)3\right] - \left[\frac{100}{2}\left(2(5)+(100-1)3\right)\right]$

$= 60,700 - 15,350$

$= 45,350$

3i. $y = \sqrt{x-3}$

x	y
3	0
3.5	$\sqrt{0.5}$
4	1
4.5	$\sqrt{1.5}$
5	$\sqrt{2}$

$h = \frac{5-3}{4} = \frac{1}{2}$

Area $= \frac{1}{2}\cdot\frac{1}{2}\left\{(0+\sqrt{2}) + 2(\sqrt{0.5}+1+\sqrt{1.5})\right\}$

$= 1.82$ (3 s.f.)

3b. Tops of the trapezia below the curve $\therefore$ underestimate

4a.
$$5^{x-1} = 120$$
$$\log(5^{x-1}) = \log 120$$
$$(x-1)\log 5 = \log 120$$
$$x-1 = \frac{\log 120}{\log 5}$$
$$x = 3.97 \ (3\text{ s.f.})$$

4b.
$$\log_2 x + 2\log_2 3 = \log_2(x+5)$$
$$\log_2 x + \log_2 9 = \log_2(x+5)$$
$$\log_2 9x = \log_2(x+5)$$
$$9x = x+5$$
$$8x = 5$$
$$x = 5/8$$

5a.
$$\frac{a}{1-r} = 4a \quad (\text{Since } S_\infty = 4 \text{ times the first term})$$
$$a = 4a(1-r) \quad (\div a)$$
$$1 = 4 - 4r$$
$$4r = 3$$
$$r = 3/4$$

5ii.
$$u_3 = ar^2$$
$$\therefore \ 9 = a\left(\frac{3}{4}\right)^2 \quad (\div (3/4)^2)$$
$$a = 16$$

5iii.
$$S_{20} = \frac{a(1-r^{20})}{1-r}$$
$$= \frac{16(1-(3/4)^{20})}{1-3/4}$$
$$= 63.8 \ (3\text{ s.f.})$$

6a. $\int \frac{x^3 + 3x^{1/2}}{x} \, dx = \int x^2 + 3x^{-1/2} \, dx$

$= \frac{1}{3}x^3 + 6x^{1/2} + c$

6bi. $\int_2^a 6x^{-4} \, dx$

$= \left[-2x^{-3} \right]_2^a$

$= -\frac{2}{a^3} - \left(-\frac{2}{2^3} \right)$

$= \frac{1}{4} - \frac{2}{a^3}$

6bii. as $a \to \infty$, $\frac{2}{a^3} \to 0$

$\therefore \int_2^\infty 6x^{-4} \, dx = \frac{1}{4} - 0$

$= \frac{1}{4}$

7i. $3\tan 2x = 1 \qquad 0 \leq x \leq 180°$

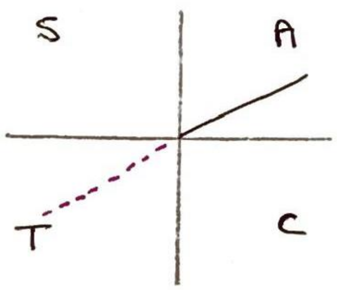
$\tan 2x = 1/3$

let $\phi = 2x \qquad 0 \leq \phi \leq 360°$

$\tan \phi = 1/3$

$\phi = \tan^{-1}(1/3)$

P.V. $\phi = 18.43°$



$\phi = 18.43, \ 198.43°$

$\therefore x = 9.2° \quad 99.2°$

7ii. $3\cos^2 x + 2\sin x - 3 = 0 \qquad 0 \leq x \leq 180^\circ$

Use $\sin^2\theta + \cos^2\theta \equiv 1 \quad \forall\ \theta \in \mathbb{R}$

$3(1 - \sin^2 x) + 2\sin x - 3 = 0$

$3 - 3\sin^2 x + 2\sin x - 3 = 0$

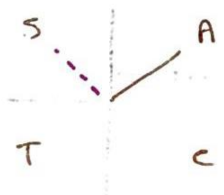
$3\sin^2 x - 2\sin x = 0$

$\sin x(3\sin x - 2) = 0$

$\sin x = 0$ or $3\sin x = 2$

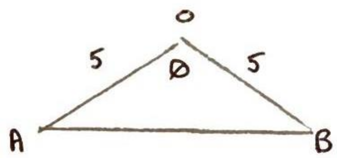
P.V. $x = 0^\circ$ $\qquad \sin x = 2/3$

P.V. $= 41.81$



$x = 0^\circ, 180^\circ, 41.81^\circ, 138.19^\circ$

8i.



Area of $\Delta = 8$

$\therefore \quad \frac{1}{2}ab\sin C = 8$

$\frac{1}{2}(5)(5)\sin\theta = 8$

$25\sin\theta = 16$

$\sin\theta = 16/25$

P.V. $\theta = 0.694$

θ is obtuse $\therefore \pi - 0.694$

$\theta = 2.45^c$ (3 s.f.)

8ii. Area of Segment $=$ Area of Sector $-$ Area of Δ

$= \frac{1}{2}r^2\theta - 8$

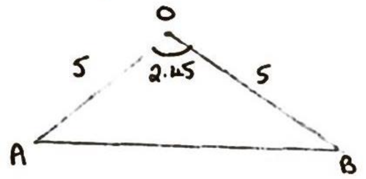
$= \frac{1}{2}(5)^2(2.45) - 8$

$= 22.6\text{ cm}^2$ (3 s.f.)

8iii. Perimeter of Segment = Arc length + length of Δ

Arc $= r\theta = 5(2.45) = 12.2\cancel{8}354\ldots$

Δ length



$AB^2 = 5^2 + 5^2 - 2(5)(5)\cos(2.45)$

$AB = 9.403$

NB. although θ quoted to 3 s.f. exact value was saved on calculator and used for calculations.

$\therefore$ Perimeter $= 9.403 + 12.235\ldots$

$= 21.6\text{cm}$ (3 s.f.)

9i. $f(x) = -4x^3 + 9x^2 + 10x - 3$

at $(3,0)$ $x = 0$ $\therefore$ $f(3) = -4(3)^3 + 9(3)^2 + 10(3) - 3$

$= -108 + 81 + 30 - 3$

$= 0$

$\therefore$ $x-3$ is a factor of $f(x)$

9ii.

$$
\begin{array}{r|l}
 & -4x^2 - 3x + 1 \\
x-3 & -4x^3 + 9x^2 + 10x - 3 \\
 & \underline{-4x^3 + 12x^2} \\
 & \quad -3x^2 + 10x \\
 & \quad \underline{-3x^2 + 9x} \\
 & \qquad x - 3 \\
 & \qquad \underline{x - 3} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore$ $f(x) = (x-3)(-4x^2 - 3x + 1)$

9iii. $-4x^2 - 3x + 1 = 0$

$4x^2 + 3x - 1 = 0$

$(4x - 1)(x + 1) = 0$

$x = 1/4$ and $x = -1$ $\therefore$ points at $(1/4, 0)$, $(-1, 0)$

9iii.

$$\int_{-1}^{1/4} -4x^3 + 9x^2 + 10x - 3 \quad dx$$

$$\left[-x^4 + 3x^3 + 5x^2 - 3x \right]_{-1}^{1/4}$$

$$F[1/4] = -\frac{101}{256}$$

$$F[-1] = 4$$

$$\text{Area} = -\frac{101}{256} - 4 = -\frac{1125}{256}$$

$$\int_{1/4}^{3} -4x^3 + 9x^2 + 10x - 3 \quad dx$$

$$\left[-x^4 + 3x^3 + 5x^2 - 3x \right]_{1/4}^{3}$$

$$F[3] = 36$$

$$F[1/4] = -\frac{101}{256}$$

$$\text{Area} = 36 - \left(-\frac{101}{256}\right) = \frac{9317}{256}$$

$$\therefore \text{ Shaded Area} = \frac{9317}{256} + \frac{1125}{256}$$

$$= \frac{5221}{128}$$