OCR

A Level

A Level Maths

OCR Core Maths C1 January 2011 Model Solutions

Name:

M M E

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Total Marks:

OCR - Jan 11 - C1

1i. A (6,1) B(-2,7)

$|AB| = \sqrt{(6--2)^2+(1-7)^2} = \sqrt{64+36} = 10$

1ii. $\text{grad} = \frac{1-7}{6--2} = -\frac{3}{4}$

1iii. $4x + 3y - 10 = 0$

$3y = 4x - 10$

$\text{grad} = \frac{4}{3}$

$\frac{4}{3} \times -\frac{3}{4} = -1$ $\therefore$ perpendicular (negative reciprocals)

2. $(x-p)(2x^2+9x+10) = (x^2-4)(2x+q)$

LHS: $(x-p)(x+2)(2x+5)$

RHS: $(2x+q)(x+2)(x-2)$

Comparing terms: $q=5$, $p=2$

3i. $\sqrt{8} = 8^{1/2}$

3ii. $\frac{1}{64} \cdot \frac{1}{8^2} = 8^{-2}$

3iii. $2^6 \times 2^2$ $\qquad 2 = 8^{1/3}$

$= (8^{\frac{1}{3}})^6 \times (8^{\frac{1}{3}})^2$

$= 8^2 \times 8^{2/3}$

$= 8^{2+\frac{2}{3}}$

$= 8^{8/3}$

4. $(3x-2)^4 - 5(3x-2)^2 + 4 = 0$

$u = (3x-2)^2$

$u^2 = (3x-2)^4$

$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 1$ or $u = 4$

$u = 1$; $(3x-2)^2 = 1$

$3x - 2 = \pm 1$

$3x = 2 \pm 1$

$x = \frac{2 \pm 1}{3}$

$x = 1$ or $\frac{1}{3}$

$u = 4$; $(3x-2)^2 = 4$

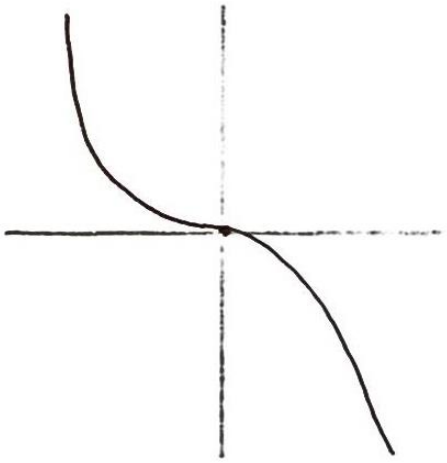
$3x - 2 = \pm 2$

$3x = 2 \pm 2$

$x = \frac{2 \pm 2}{3}$

$x = \frac{4}{3}$ or 0

5.



$y = -x^3$

5ii. $-x^3 \rightarrow -(x-3)^3$ (Since $f(x) \rightarrow f(x-a)$ is a translation a units in the positive x direction)

$y = -(x-3)^3$

5iii. $-x^3 \rightarrow -5x^3$ Stretch, s.f. 5 in the y direction

6i. $y = \frac{5}{x^2} - \frac{1}{4x} + x$

$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$

$\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$

6ii. $\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$

7i. $4x^2 + 12x - 3$

$4\left[x^2 + 3x\right] - 3$

$4\left[(x+3/2)^2 - (\frac{3}{2})^2\right] - 3$

$4(x+3/2)^2 - 4(\frac{9}{4}) - 3$

$4(x+3/2)^2 - 12$

7ii. $4(x+3/2)^2 - 12 = 0$

$(x+3/2)^2 = 3$

$x + 3/2 = \pm\sqrt{3}$

$x = -3/2 \pm \sqrt{3}$

7iii. $4x^2 + 12x - k = 0$

equal roots $\therefore$ disc $= 0$

$12^2 - 4(4)(-k) = 0$

$144 + 16k = 0$

$16k = -144$

$k = -9$

8i. $y = 7 + 6x - x^2$

at P, $x = 5$, $y = 7 + 6(5) - (5)^2$

$= 7 + 30 - 25$

$= 12$

so P at $(5, 12)$

$\frac{dy}{dx} = 6 - 2x$

at P, $\frac{dy}{dx} = 6 - 2(5)$

$= 6 - 10$

$= -4$

$y - 12 = -4(x - 5)$

$y - 12 = -4x + 20$

$4x + y - 32 = 0$

8ii. at Q, $y = 0$ $\therefore$ $4x + 0 - 32 = 0$

$4x = 32$

$x = 8$

so Q at $(8, 0)$

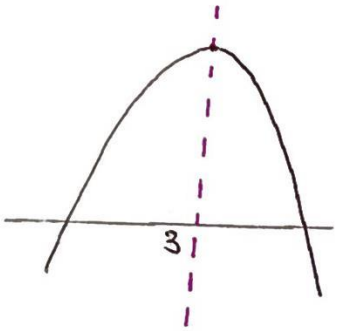
Mid PQ $= \left(\frac{5+8}{2}, \frac{12+0}{2}\right) = \left(\frac{13}{2}, 6\right)$

8iii. Symmetrical about stat. point

$\frac{dy}{dx} = 0 \Rightarrow 2x = 6$

$x = 3$

symmetrical about the line $x = 3$



8iv. Increasing when $\frac{dy}{dx} > 0$

$\therefore$ $6 - 2x > 0$

$6 > 2x$

$x < 3$

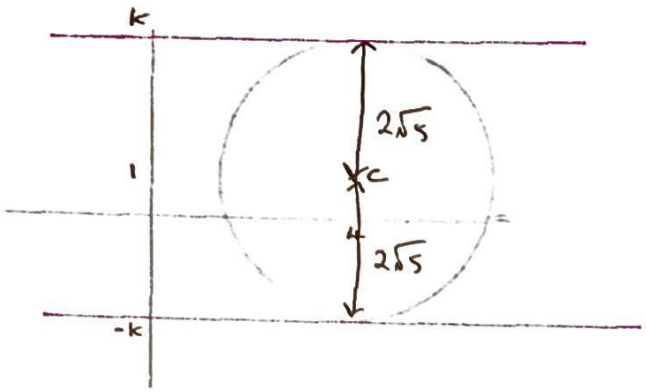
9i. $x^2 + y^2 - 8x - 2y - 3 = 0$

$(x-4)^2 - 16 + (y-1)^2 - 1 - 3 = 0$

$(x-4)^2 + (y-1)^2 = 20$

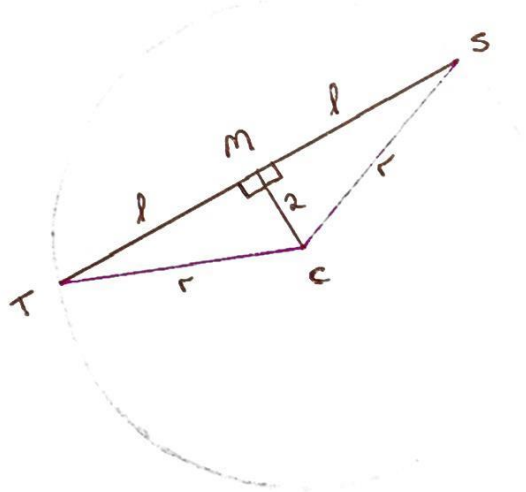
Centre at $(4,1)$, Radius $\sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$

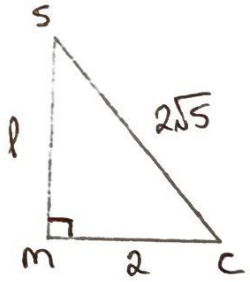
9ii.



$y = k$ is tangent when $k = 1 \pm 2\sqrt{5}$

9iii.





$l^2 + 2^2 = (2\sqrt{5})^2$

$l^2 = 16$

$l = 4$

$ST = 2l$

$= 2(4)$

$= 8$

9iv. $(x-4)^2 + (y-1)^2 = 20$ ①

$x - 2y - 12 = 0 \Rightarrow x = 2y + 12$ ②

'Sub ② into ①'

$(2y+12-4)^2 + (y-1)^2 = 20$

$(2y+8)^2 + (y-1)^2 = 20$

$4y^2 + 32y + 64 + y^2 - 2y + 1 = 20$

$5y^2 + 30y + 45 = 0$

$y^2 + 6y + 9 = 0$

$(y+3)^2 = 0$

$y = -3$

when $y = -3$; $x = 2(-3) + 12$

$= -6 + 12$

$= 6$

$\therefore$ circle meets the line at $(6, -3)$