OCR

A Level

A Level Maths

OCR Core Maths C2 January 2010 Model Solutions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks:

OCR Jan 10 C2

1i. $2\sin^2 x = 5\cos x - 1$

USE $\sin^2 x \equiv 1 - \cos^2 x, \quad \forall x \in \mathbb{R}$

$2(1-\cos^2 x) = 5\cos x - 1$

$2 - 2\cos^2 x = 5\cos x - 1$

$2\cos^2 x + 5\cos x - 3 = 0$

1ii. $(2\cos x - 1)(\cos x + 3) = 0$

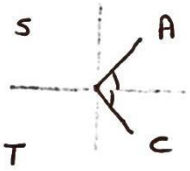
$2\cos x = 1$ or $\cos x = -3$ ✗

$\cos x = 1/2$

$x = \cos^{-1}(1/2)$

P.V. $= 60°$

$x = 60°, 300°$



2i. $\frac{dy}{dx} = 6x - 4$

$y = \int 6x - 4\,dx$

$= 3x^2 - 4x + c$

when $x = 2, \ y = 5 \Rightarrow \quad 5 = 3(2)^2 - 4(2) + c$

$c = 1$

$\therefore \quad y = 3x^2 - 4x + 1$

2ii. $x = p, \ y = 5 \Rightarrow \quad 5 = 3p^2 - 4p + 1$

$3p^2 - 4p - 4 = 0$

$(3p+2)(x-2) = 0$

$p = 2$ or $-2/3 \quad \therefore \quad p = -2/3$

3i. $(2-x)^7 = 2^7 + {}^7C_1 2^6(-x) + {}^7C_2 2^5(-x)^2 + {}^7C_3 2^4(-x)^3$

$= 128 - 448x + 672x^2 - 560x^3$

3ii. $\left(2 - \frac{1}{4}w^2\right)^7$

$\therefore \quad x = \frac{1}{4}w^2$

$x^3 = \frac{1}{64}w^6$

$\therefore \quad -560 \cdot \frac{1}{64} w^6$

coefficient of w^6 : $\frac{-560}{64} = -\frac{35}{4}$

4i. $\int_3^5 \log(2+x)\,dx \qquad h = \frac{5-3}{4} = 0.5$

x	y
3	$\log 5$
3.5	$\log 5.5$
4	$\log 6$
4.5	$\log 6.5$
5	$\log 7$

$\int \approx \frac{0.5}{2}\left\{(\log 5 + \log 7) + 2(\log 5.5 + \log 6 + \log 6.5)\right\}$

$= 1.551730659\ldots$

$= 1.55$ to 3 s.f.

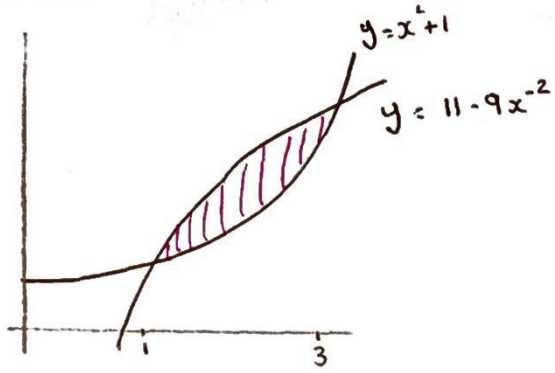
4ii. $\log\sqrt{2+x} = \log(2+x)^{1/2} = \frac{1}{2}\log(2+x)$

$\int_3^5 \log\sqrt{2+x}\,dx = \frac{1}{2}\int_3^5 \log(2+x)\,dx$

$= \frac{1}{2} \times 1.55$

$= 0.78$ (2 s.f.)

5.



Shaded $= \int_1^3 11 - 9x^{-2}\,dx - \int_1^3 x^2+1\,dx$

$\left[11x + 9x^{-1}\right]_1^3$ $\qquad$ $\left[\frac{1}{3}x^3 + x\right]_1^3$

$F[3] = 11(3) + 9(3)^{-1}$ $\qquad$ $F[3] = \frac{1}{3}(3)^3 + 3$

$= 36$ $\qquad$ $= 12$

$F[1] = 11(1) + 9(1)^{-1}$ $\qquad$ $F[1] = \frac{1}{3} + 1$

$= 20$ $\qquad$ $= 4/3$

$\int = 16$ $\qquad$ $\int = 32/3$

$\therefore$ Shaded area $: 16 - 32/3 = 16/3$

6i. $f(x) = 2x^3 + ax^2 + bx + 15$

$(x+3)$ is a factor $\therefore f(-3) = 0$

$f(-3) = 2(-3)^3 + a(-3)^2 + b(-3) + 15$

$0 = -54 + 9a - 3b + 15$

$3a - b = 13$ $\quad$ ①

$f(2) = 35$

$f(2) = 2(2)^3 + a(2)^2 + b(2) + 15$

$16 + 4a + 2b + 15 = 35$

$2a + b = 2$ $\quad$ ②

'① + ②' $\quad 5a = 15$

$a = 3 \qquad b = -4$

6ii.

$$x+3 \overline{)\,2x^3+3x^2-4x+15}$$

$$2x^2-3x+5$$

$$\begin{array}{r} 2x^3+6x^2 \\ \hline -3x^2-4x \\ -3x^2-9x \\ \hline 5x+15 \\ 5x+15 \\ \hline 0 \end{array}$$

$\therefore \quad f(x) = (x+3)(2x^2-3x+5)$

7i.

$a^2 = b^2+c^2-2bc\cos A$

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{14^2+10^2-13^2}{2(14)(10)}$

$= \dfrac{127}{280}$

$A = \cos^{-1}\left(\dfrac{127}{280}\right) = 1.1000277\ldots$

$= 1.10 \quad (3\text{s.f.})$

7ii.

$EF = r\theta$

$= 4(1.10)$

$= 4.4\text{ cm}$

$\therefore$ Perimeter $= 4.4+6+13+10 = 33.4\text{ cm}$

7iii.

$\triangle ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(10)(14)\sin(1.10)$

$= 62.3845$

Sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(4)^2(1.1)$

$= 44/5$

$\therefore$ Shaded Area $= 62.3845 - 44/5$

$= 53.6\text{ cm}^2 \quad (3\text{s.f.})$

8i. $u_1 = 8$ $\quad u_{n+1} = u_n + 3$

$u_2 = 8+3 = 11$

$u_3 = 11+3 = 14$

$u_4 = 14+3 = 17$

$u_5 = 17+3 = 20$

8ii. $u_n = pn + q$

$u_1 = 8 \Rightarrow 8 = p + q$ ①

$u_5 = 20 \Rightarrow 20 = 5p + q$ ②

'② - ①' $\quad 12 = 4p \Rightarrow p = 3 \quad q = 5$

8iii. Arithmetic Sequence $(a = 8, d = 3)$

8iv. $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$

$S_{2N} = \frac{2N}{2}\left(2(8) + (2N-1)3\right) = N(6N+13)$

$S_N = \frac{N}{2}\left(2(8) + (N-1)3\right) = \frac{N}{2}(3N+13)$

$6N^2 + 13N - \frac{3N^2}{2} + \frac{13N}{2} = 1256$

$9N^2 + 13N - 2512 = 0$

$N = \frac{-13 \pm \sqrt{13^2 - 4(9)(2512)}}{18}$

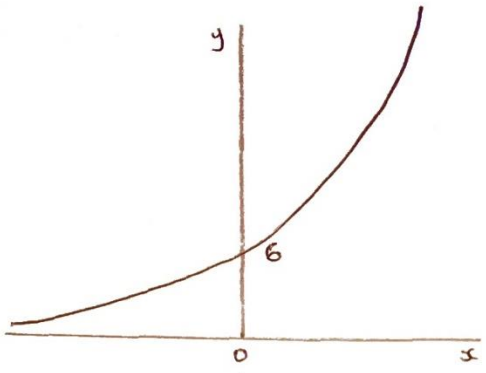
$= \frac{-13 \pm 301}{18}$

(+) $\frac{-13+301}{18} = 16$ $\qquad$ (-) $\frac{-13-301}{18} = -\frac{157}{9}$

N must be a positive integer

$\therefore \quad N = 16$

9i.



$y = 6 \times 5^x$

9ii.

$150 = 9^x$

$\log 150 = \log 9^x$

$\log 150 = x \log 9$

$x = \dfrac{\log 150}{\log 9}$

$= 2.28 \quad (3 \text{ s.f.})$

9iii.

$6 \times 5^x = 9^x$

$\log_3(6 \times 5^x) = \log_3(9^x)$

$\log_3 6 + \log_3 5^x = \log_3 9^x$

$\log_3 6 + x \log_3 5 = x \log_3 9$

$\log_3 6 = \log_3 3 + \log_3 2$

$= 1 + \log_3 2$

$\log_3 9 = 2$

$\therefore \quad 1 + \log_3 2 + x \log_3 5 = 2x$

$2x - x \log_3 5 = 1 + \log_3 2$

$x(2 - \log_3 5) = 1 + \log_3 2$

$x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$