OCR

A Level

A Level Maths

OCR Core Maths C1 January 2010 Model Solutions

Name:

MME
Mathsmadeeasy.co.uk

Total Marks:

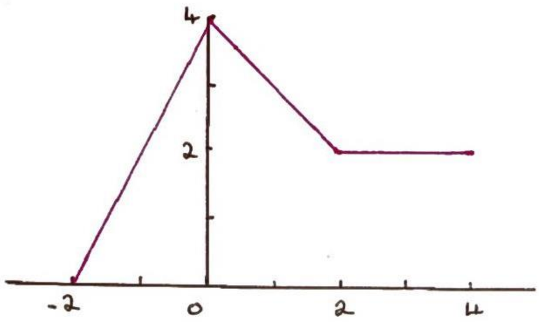
OCR - Jan 10 C1

1. $x^2 - 12x + 1$

$(x-6)^2 - 36 + 1$

$(x-6)^2 - 35$

2i.



$f(x) \rightarrow 2f(x)$

stretch s.f. 2 in y direction

2ii. $f(x) \rightarrow f(x-1)$ translation 1 unit in the positive x direction

3. $y = x^3 - 4x^2 + 7$ $\quad (2, -1)$

$\frac{dy}{dx} = 3x^2 - 8x$

at $(2,-1)$, $\frac{dy}{dx} = 3(2)^2 - 8(2)$

$= -4$

$\therefore$ grad of normal $= 1/4$

$(y+1) = \frac{1}{4}(x-2)$ $\quad \times 4$

$4y + 4 = x - 2$

$x - 4y - 6 = 0$

4i. $3^m = 81$; $m = 4$

4ii. $(36p^4)^{1/2} = 24$

$6p^2 = 24$

$p^2 = 4$

$p = \pm 2$

4iii. $5^n \times 5^{n+4} = 25$

$5^{2n+4} = 5^2$

$\therefore \quad 2n + 4 = 2$

$n = -1$

5. $x - 8\sqrt{x} + 13 = 0$ Let $y = \sqrt{x}$

$y^2 - 8y + 13 = 0$ $y^2 = x$

$(y-4)^2 - 16 + 13 = 0$

$(y-4)^2 = 3$

$y = 4 \pm \sqrt{3}$

$\sqrt{x} = 4 \pm \sqrt{3}$

$x = (4 \pm \sqrt{3})^2$

$= 16 \pm 8\sqrt{3} + 3$

$= 19 \pm 8\sqrt{3}$

6i. $y = x^2 + 5$

$\frac{dy}{dx} = 2x$

at $(1,6)$ $\frac{dy}{dx} = 2(1) = 2$

6ii. grad $= \frac{y - y_1}{x - x_1}$ $\therefore \quad \frac{a^2 + 5 - 6}{a - 1} = 2.3$

$\frac{a^2 - 1}{a - 1} = 2.3$

$\frac{(a+1)\cancel{(a-1)}}{\cancel{(a-1)}} = 2.3$

$a + 1 = 2.3$

$a = 1.3$

6iii. $2 < \text{grad.} < 2{\cdot}3$ (Since grad. of line segment is steeper)

e.g. $2{\cdot}1$

7ia. $y = (3-x)^2$, double root at $x = 3$

positive x^2 $\therefore$ $\cup$ shaped

$\therefore$ Figure 3

7ib. $y = x^2 + 9$, positive x^2, moved up 9

$\therefore$ Figure 1

7ic. $y = (3-x)(x+3)$, roots at $x = \pm 3$

negative x^2 $\therefore$ $\cap$ shaped

$\therefore$ Figure 4

7ii. Figure 2 is an x^2 parabola translated 3 units right, and reflected in the x axis.

$x^2 \longrightarrow (x-3)^2$ translation

$(x-3)^2 \longrightarrow -(x-3)^2$ reflection

$\therefore \quad y = -(x-3)^2$

8i. $x^2 + y^2 + 6x - 4y - 4 = 0$

$(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$

$(x+3)^2 + (y-2)^2 = 17$

$\therefore$ centre at $(-3, 2)$

radius $= \sqrt{17}$

8ii. $(x+3)^2 + (y-2)^2 = 17$ ①

$y = 3x+4$ ②

To find where the line meets the curve, solve simultaeneously

'sub ② into ①'

$(x+3)^2 + (3x+4-2)^2 = 17$

$(x+3)^2 + (3x+2)^2 = 17$

$x^2 + 6x + 9 + 9x^2 + 12x + 4 = 17$

$10x^2 + 18x - 4 = 0$

$5x^2 + 9x - 2 = 0$

$(5x-1)(x+2) = 0$

$x = -2$ or $x = 1/5$

$x = -2$; $y = 3(-2) + 4$
$= -2$ $\therefore$ point at $(-2, -2)$

$x = 1/5$; $y = 3(\frac{1}{5}) + 4$
$= \frac{23}{5}$ $\therefore$ point at $(\frac{1}{5}, \frac{23}{5})$

9a. $f(x) = \frac{1}{x} - \sqrt{x} + 3$

$= x^{-1} - x^{1/2} + 3$

$f'(x) = -x^{-2} - \frac{1}{2}x^{-1/2}$

9ii. $f''(x) = 2x^{-3} + \frac{1}{4}x^{-3/2}$

$= \frac{2}{x^3} + \frac{1}{4.(\sqrt{x})^3}$

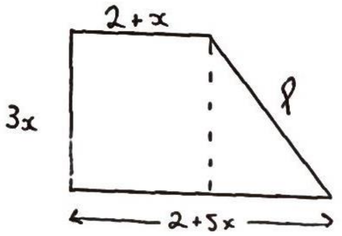
$f''(4) = \frac{2}{4^3} + \frac{1}{4.(\sqrt{4})^3} = \frac{1}{32} + \frac{1}{32} = \frac{1}{16}$

10. $kx^2 - 30x + 25k = 0$

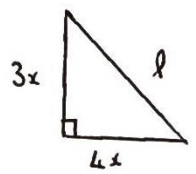
equal roots, $\therefore$ disc. $= 0$

$$(-30)^2 - 4k(25k) = 0$$
$$900 = 100k^2$$
$$9 = k^2$$
$$k = \pm 3$$

11i.



To find l:



$$l^2 = (3x)^2 + (4x)^2$$
$$= 25x^2$$
$$\therefore \ l = 5x$$

$$P = 5x + 2 + x + 3x + 2 + 5x$$
$$= 14x + 4$$

11ii. Area of trapezium $= \frac{1}{2}(a+b)h$

$$= \frac{1}{2}(2+x+2+5x)3x$$
$$= 3x(3x+2)$$
$$= 9x^2 + 6x$$

11iii. Perimeter : $14x + 4 \geq 39$

$$14x \geq 35$$
$$2x \geq 5$$
$$x \geq 2.5$$

Area : $9x^2 + 6x < 99$

$$3x^2 + 2x - 33 < 0$$
$$(3x+11)(x-3) < 0$$

$\therefore \ -\frac{11}{3} < x < 3$ but x must be positive

$0 < x < 3$

$\therefore \ \underline{2.5 \leq x < 3}$