Edexcel

A Level

A Level Maths

Edexcel Core Maths C4 June 2014 Model Solutions

Name:

MME
Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 14 C4

1a. $x^3 + 2xy - x - y^3 - 20 = 0$

$3x^2 + 2y + 2x\frac{dy}{dx} - 1 - 3y^2\frac{dy}{dx} = 0$

$3x^2 + 2y - 1 = \frac{dy}{dx}(3y^2 - 2x)$

$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$

1b. at C, $x = 3$, $y = -2$

$\frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)} = \frac{11}{3}$

$y - -2 = \frac{11}{3}(x-3)$

$y + 2 = \frac{11}{3}(x-3)$

$3y + 6 = 11x - 33$

$11x = 3y + 39$

2a. $(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-5)}{2}(kx)^2 + \ldots$

$= 1 - 4kx + 10k^2x^2 + \ldots$

$-4k = -6$

$k = 3/2$

2b. $10k^2 = A$

$10\left(\frac{3}{2}\right)^2 = A$

$10 = A/(3/2)^2 \Rightarrow A = 90/4$

3a $x = 3 \Rightarrow y = 0.68212$

3b. $h = 1, \quad \int \approx \frac{1}{2}\left\{(1.42857 + 0.55556 + 2(0.90326 + 0.68212)\right\}$

$= 2.5774 \quad (4\text{dp.})$

3c. overestimate since the tops of trapezia lie above the curve

3d. $\int_1^4 \frac{10}{2x + 5\sqrt{x}} \, dx$

$u = x^{1/2}$

$x = u^2$

$dx = 2u \, du$

x	4	1
u	2	1

$= \int_1^2 \frac{10}{2u^2 + 5u} \cdot 2u \, du$

$= \int_1^2 \frac{20u}{u(2u+5)} \, du$

$= 20 \int_1^2 \frac{1}{2u+5} \, du$

$= 10\left[\ln|2u+5|\right]_1^2$

$= 10(\ln 9 - \ln 7)$

$= 10 \ln(9/7)$

4. $\frac{dv}{dt} = 80\pi$

$V = 4\pi h^2 + 16\pi h$

$\frac{dV}{dh} = 8\pi h + 16\pi$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dv} \quad = \quad \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$

$= \frac{10}{h+2}$

when $h = 6, \quad \frac{dh}{dt} = \frac{10}{6+2} = 1.25$

5a. $x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leq t < 2\pi$

$$x + y = 4\cos(t + \pi/6) + 2\sin t$$
$$= 4(\cos t \cos \pi/6 - \sin t \sin \pi/6) + 2\sin t$$
$$= 4\left(\frac{\sqrt{3}}{2}\cos t - \frac{1}{2}\sin t\right) + 2\sin t$$
$$= 2\sqrt{3}\cos t - 2\sin t + 2\sin t$$
$$= 2\sqrt{3}\cos t$$

5b.

$$x + y = 2\sqrt{3}\cos t$$
$$(x+y)^2 = (2\sqrt{3}\cos t)^2$$
$$= 12\cos^2 t$$
$$= 12(1 - \sin^2 t)$$
$$= 12 - 12\sin^2 t$$

$$y = 2\sin t$$
$$y^2 = 4\sin^2 t$$
$$3y^2 = 12\sin^2 t$$

$$\therefore (x+y)^2 + 3y^2 = 12$$

6ai. $\int x e^{4x}\, dx$

Parts $\quad u = x \quad u' = 1 \qquad v' = e^{4x} \quad v = \frac{1}{4}e^{4x}$

$$= \frac{x}{4}e^{4x} - \int \frac{1}{4}e^{4x}\, dx$$

$$= \frac{x}{4}e^{4x} - \frac{1}{16}e^{4x} + c$$

6ii.

$$\int \frac{8}{(2x-1)^3}\,dx$$

$$\frac{d}{dx}\left(k(2x-1)^{-2}\right) = 8(2x-1)^{-3}$$

$$-2k.2.(2x-1)^{-3} = 8(2x-1)^{-3}$$

$$-4k = 8 \quad \Rightarrow \quad k = -2$$

$$\int \frac{8}{(2x-1)^3}\,dx = -2(2x-1)^{-2} + c$$

6iii.

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

$$\frac{dy}{dx} = e^x . \frac{1}{\sin 2y} . \frac{1}{\sin y}$$

$$\int \sin 2y . \sin y \; dy = \int e^x \, dx$$

$$\int 2\sin y \cos y . \sin y \; dy = \int e^x \, dx \qquad (\sin 2y = 2\sin y \cos y)$$

$$\int 2\sin^2 y \cos y \; dy = \int e^x \, dx$$

$$\frac{d}{dx}\left(k(\sin y)^3\right) = 2\cos y(\sin y)^2$$

$$3k\cos y(\sin y)^2 = 2\cos y(\sin y)^2$$

$$3k = 2$$

$$k = 2/3$$

$$\frac{2}{3}\sin^3 y = \int e^x \, dx$$

$$\frac{2}{3}\sin^3 y = e^x + c$$

$y = \pi/6$ when $x = 0$ $\Rightarrow$ $\frac{1}{12} = 1 + c$ $\qquad c = -\frac{11}{12}$

$$\therefore \quad \frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$$

7a. $x = 3\tan\theta$, $y = 4\cos^2\theta$ $\quad 0 \leq \theta < \frac{\pi}{2}$

$\frac{dx}{d\theta} = 3\sec^2\theta \qquad \frac{dy}{dx} = -8\cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$$

$$= -\frac{8}{3}\sin\theta\cos^3\theta$$

when $x = 3$, $\quad 3\tan\theta = 3$

$\tan\theta = 1$

$\theta = \pi/4$

$$\frac{dy}{dx} = -\frac{8}{3}\cos^3(\pi/4)\sin(\pi/4)$$

$$= -\frac{2}{3}$$

$\therefore$ m of normal $= \frac{3}{2}$ (since $\perp$)

$y - 2 = \frac{3}{2}(x-3)$

$2y - 4 = 3x - 9$

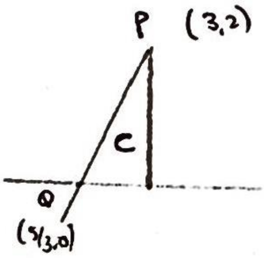
crosses x axis when $y = 0$

$-4 = 3x - 9$

$3x = 5$

$x = 5/3$

7b.



Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(2)^2\left(\frac{4}{3}\right)$$

$$= \frac{16\pi}{9}$$

$$V = \pi \int_0^3 y^2 \, dx$$

$$y = 4\cos^2\theta$$

$$y^2 = 16\cos^4\theta$$

x	3	0
θ	$\pi/4$	0

$$= \pi \int_0^{\cancel{\pi/3}\ \pi/4} 16\cos^4\theta \ \frac{dx}{d\theta} \ d\theta$$

$$= \pi \int_0^{\pi/4} 16\cos^4\theta \, . \, 3\sec^2\theta \ d\theta$$

$$= 48\pi \int_0^{\pi/4} \cos^2\theta \ d\theta$$

$$\left(\cos^2\theta \equiv \tfrac{1}{2}\cos 2\theta + \tfrac{1}{2}\right)$$

$$= 48\pi \int_0^{\pi/4} \tfrac{1}{2}\cos 2\theta + \tfrac{1}{2} \ d\theta$$

$$= 48\pi \left[\tfrac{1}{4}\sin 2\theta + \tfrac{1}{2}\theta \right]_0^{\pi/4}$$

$$= 48\pi \left(1/4 + \tfrac{1}{8}\pi \right)$$

$$= 12\pi + 6\pi^2$$

Shaded Area $= 12\pi + 6\pi^2 - \frac{16}{9}\pi$ (cone)

$$= \frac{92}{9}\pi + 6\pi^2$$

8a. $A \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$ $\quad B \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

8b. $\underline{r} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

8c. $\overrightarrow{BA} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ $\quad \overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

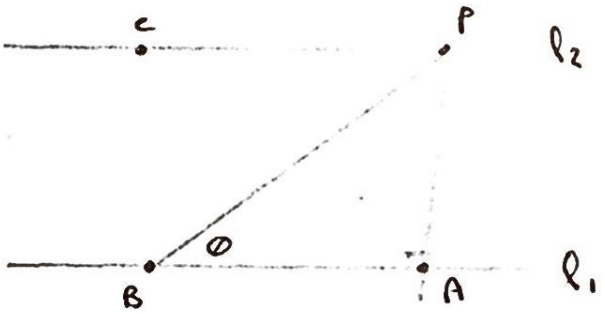
$$= \frac{-1(1) + 1(-1) + -1(-5)}{\sqrt{1^2+1^2+1^2} \times \sqrt{1^2+1^2+5^2}}$$

$$\cos\theta = \frac{3}{\sqrt{3}.\sqrt{27}}$$

$$= \frac{1}{3}$$

8d. l_2 $\quad \underline{r} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

8e.



$\overrightarrow{PC} = \overrightarrow{AB}$ so C at P + $\overrightarrow{AB}$

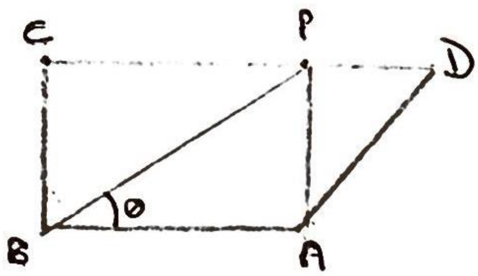
$$\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

$\overrightarrow{PD} = \overrightarrow{BA}$ so D at P + $\overrightarrow{BA}$

$$\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

C (1, 1, 4) $\quad$ D (-1, 3, 2)

8f.

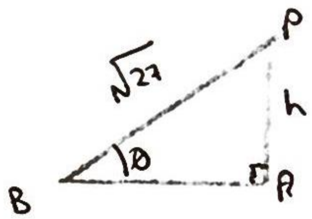


Area $= \frac{1}{2}\left(|CD| + |BA|\right) \times |PA|$

$|CD| = 2|AB|$

$|AB| = \sqrt{1^2+1^2+1^2} = \sqrt{3} \Rightarrow |CD| = 2\sqrt{3}$

$|PB| = \sqrt{27}$



$\cos\theta = \frac{1}{3}$

$\sin\theta = \frac{h}{\sqrt{27}}$

$\therefore \quad h = 2\sqrt{6}$

Area $= \frac{1}{2}\left(2\sqrt{3} + \sqrt{3}\right) \times 2\sqrt{6}$

$= 9\sqrt{2}$