Edexcel

A Level

A Level Maths

Edexcel Core Maths C3 June 2014 Model Solutions

Name:

Mathsmadeeasy.co.uk

Total Marks:

Edexcel June 14 C3

1a. $f(x) = \dfrac{4x+1}{x-2}$

$f = 4x+1$ $\quad g = x-2$

$f' = 4$ $\quad g' = 1$

$$f'(x) = \frac{4(x-2) - 1(4x+1)}{(x-2)^2}$$

$$= \frac{4x-8-4x-1}{(x-2)^2}$$

$$= \frac{-9}{(x-2)^2}$$

1b. $-1 = \dfrac{-9}{(x-2)^2}$

$(x-2)^2 = 9$

$x-2 = \pm 3$

$x = 2 \pm 3$

$x = 5$ or $x = -1$

$\therefore \quad x = 5 \quad$ (Since $x > 2$)

when $x = 5$, $\quad y = \dfrac{4(5)+1}{5-2}$

$= 7$

$\therefore \quad P$ at $(5, 7)$

2a.

$$2\ln(2x+1) - 10 = 0$$
$$\ln(2x+1) = 5$$
$$2x+1 = e^5$$
$$x = \frac{1}{2}(e^5 - 1)$$

2b.

$$3^x e^{4x} = e^7$$
$$3^x = e^{7-4x}$$
$$x\ln 3 = 7 - 4x$$
$$x(\ln 3 + 4) = 7$$
$$x = \frac{7}{\ln 3 + 4}$$

3a.

$$x = 8y\tan 2y \qquad P\left(\pi, \frac{\pi}{8}\right)$$
$$\pi = 8(\pi/8)\tan\left(2(\pi/8)\right)$$
$$\pi = \pi\tan(\pi/4)$$
$$\pi = \pi \quad \therefore P \text{ lies on } C$$

3b.

$$x = 8y\tan 2y$$
$$\frac{dx}{dy} = 8\tan 2y + 16y\sec^2 2y$$
$$\text{when } y = \pi/8, \quad \frac{dx}{dy} = 8\tan(2\pi/8) + 16\left(\tfrac{\pi}{8}\right)\sec^2(2\pi/8)$$
$$= 8 + 4\pi$$
$$\therefore \frac{dy}{dx} = \frac{1}{8+4\pi}$$

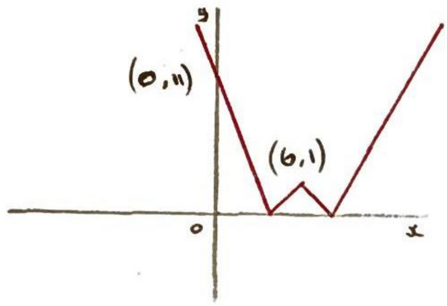
$$y - \frac{\pi}{8} = \frac{1}{8+4\pi}(x - \pi)$$

$$\left(y - \frac{\pi}{8}\right)(8 + 4\pi) = x - \pi$$

$$8y + 4\pi y - \pi - \frac{\pi^2}{2} = x - \pi$$

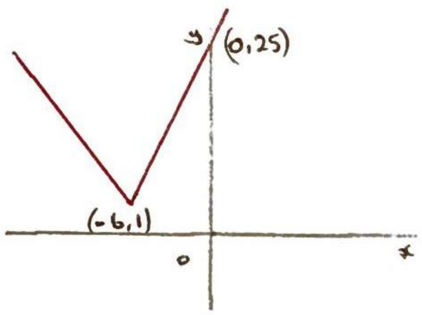
$$y(8 + 4\pi) = x + \frac{\pi^2}{2}$$

4a.



$y = |f(x)|$

4b.



$y = 2f(-x) + 3$

4c.

when $x = 6$, $f(x) = -1$ $\therefore$

$$a|6-b| - 1 = -1$$

$$a|6-b| = 0 \Rightarrow a = 0 \text{ or } b = 6$$

when $x = 0$, $f(x) = 11$ $\therefore$

$$a|-b| - 1 = 11$$

$$a|b| = 12$$

$$a \neq 0 \therefore b = 6$$

$$\Rightarrow a = 2$$

5a. $g(x) = \dfrac{x}{x+3} + \dfrac{3(2x+1)}{x^2+x-6}, \quad x > 3$

$= \dfrac{x}{x+3} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

$= \dfrac{x(x-2)}{(x-2)(x+3)} + \dfrac{3(2x+1)}{(x+3)(x-2)}$

$= \dfrac{x(x-2) + 3(2x+1)}{(x-2)(x+3)}$

$= \dfrac{x^2 - 2x + 6x + 3}{(x-2)(x+3)}$

$= \dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)}$

$= \dfrac{x+1}{x-2}$

5b. $g(x) = \dfrac{x+1}{x-2} \quad x > 3$

when $x = 3 \quad g(x) = \dfrac{4}{1} = 4$

as $x \to \infty \quad g(x) \to 1$

$\therefore \quad 1 < g(x) < 4$

5c. $g(x) = \dfrac{x+1}{x-2} \; ; \; x > 3$

let $y = \dfrac{x+1}{x-2}$

$yx - 2y = x + 1$

$yx - x = 2y + 1$

$$x(y-1) = 2y + 1$$

$$x = \frac{2y+1}{y-1}$$

$$\therefore \quad g^{-1}(x) = \frac{2x+1}{x-1}$$

$$\frac{2x+1}{x-1} = \frac{x+1}{x-2}$$

$$(2x+1)(x-2) = (x+1)(x-1)$$

$$2x^2 - 4x + x - 2 = x^2 - 1$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

since $x > 3$ $\quad x = \dfrac{3 + \sqrt{13}}{2}$

6a. $$y = 2\cos\left(\tfrac{1}{2}x^2\right) + x^3 + 3x - 2$$

when $x = 2.1$, $y = 2\cos\left(\tfrac{1}{2}(2.1)^2\right) + (2.1)^3 - 3(2.1) - 2$

$= -0.22$

when $x = 2.2$, $y = 2\cos\left(\tfrac{1}{2}(2.2)^2\right) + (2.2)^3 - 3(2.2) - 2$

$= 0.55$

change of sign $\therefore$ point lies between 2.1 and 2.2

6b. R = minimum $\Rightarrow$ at R $\dfrac{dy}{dx} = 0$

$$y = 2\cos\left(\tfrac{1}{2}x^2\right) + x^3 - 3x - 2$$

$$\frac{dy}{dx} = -2x\sin\left(\tfrac{1}{2}x^2\right) + 3x^2 - 3$$

$$0 = -2x\sin\left(\tfrac{1}{2}x^2\right) + 3x^2 - 3$$

$$2x\sin\left(\tfrac{1}{2}x^2\right) + 3 = 3x^2$$

$$\tfrac{2}{3}x\sin\left(\tfrac{1}{2}x^2\right) + 1 = x^2$$

$$x = \sqrt{1 + \tfrac{2}{3}x\sin\left(\tfrac{1}{2}x^2\right)}$$

6c.

$$x_{n+1} = \sqrt{1 + \tfrac{2}{3}x_n\sin\left(\tfrac{1}{2}x_n^2\right)}, \quad x_0 = 1.3$$

$x_1 = 1.284$ (3dp)

$x_2 = 1.276$ (3dp)

7a.

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90^\circ n, \quad n \in \mathbb{Z}$$

LHS $\dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$

$$= \frac{1 + \cos 2x}{\sin 2x}$$

$$= \frac{1 + (2\cos^2 x - 1)}{2\sin x\cos x}$$

$$= \frac{2\cos^2 x}{2\sin x\cos x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x = \text{RHS}$$

7b.

$$\text{cosec}(4\theta + 10) + \cot(4\theta + 10) = \sqrt{3} \qquad 0 \leq \theta < 180$$

Let $4\theta + 10 = 2x$

$2\theta + 5 = x$

$\therefore$

$$\cot(2\theta + 5) = \sqrt{3}$$

$$\tan(2\theta + 5) = \frac{1}{\sqrt{3}}$$

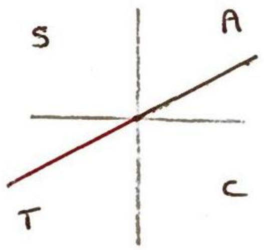
Let $\phi = 2\theta + 5$

$$\tan \phi = \frac{1}{\sqrt{3}}$$

$0 \leq 2\theta < 360$

$5 \leq \phi < 365$

P.V. $\phi = 30°$



$\phi = 30°, 210°$

$2\theta + 5 = 30° \Rightarrow \theta = 12.5°$

$2\theta + 5 = 210° \Rightarrow \theta = 102.5°$

8a.

$t = 0 \quad \therefore \quad P = \dfrac{800e^0}{1 + 3e^0}$

$= 200$

8b.

$$250 = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}$$

$$250 + 750e^{0.1t} = 800e^{0.1t}$$

$$250 = 50e^{0.1t}$$

$$5 = e^{0.1t}$$

$$\ln 5 = 0.1t$$

$$t = \frac{\ln 5}{0.1} = 10 \ln 5$$

8c.

$P = \dfrac{800e^{0.1t}}{1+3e^{0.1t}}$ $\quad f = 800e^{0.1t}$ $\quad g = 1+3e^{0.1t}$

$f' = 80e^{0.1t}$ $\quad g' = 0.3e^{0.1t}$

$$\frac{dP}{dt} = \frac{80e^{0.1t}(1+3e^{0.1t}) - 800e^{0.1t}(0.3e^{0.1t})}{(1+3e^{0.1t})^2}$$

$$= \frac{80e^{0.1t} + 240e^{0.1t} - 240e^{0.1t}}{1 + 6e^{0.1t} + 9e^{0.2t}}$$

when $t=10$, $\dfrac{dP}{dt} = \dfrac{80e}{1+6e+9e^2}$

8d.

as $t \to \infty$, $\cancel{\dfrac{dP}{dt}}$ $\dfrac{800e^{0.1t}}{1+3e^{0.1t}} \to \dfrac{800}{3}$

$\dfrac{800}{3} < 270$ $\quad \therefore$ P will never reach 270

9a.

$2\sin\theta - 4\cos\theta = R\sin(\theta - \alpha)$

$= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

$\sin\theta$: $2 = R\cos\alpha$ ①

$\cos\theta$; $-4 = -R\sin\alpha$

$4 = R\sin\alpha$ ②

'② ÷ ①' $\quad \tan\alpha = 2$

$\alpha = 1.107^c$

$R = \sqrt{2^2+4^2}$

$= 2\sqrt{5}$

$2\sin\theta - 4\cos\theta = 2\sqrt{5}\sin(\theta - 1.107)$

9bi.

$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$

$= 4 + 5(2\sqrt{5}(\sin(3\theta - 1.107))^2$

max value of $\sin(3\theta - 1.107) = 1$

$H(\theta) = 4 + 5(2\sqrt{5} \times 1)^2$

$= 104$

9bii.

$\sin(3\theta - 1.107) = 1$

let $\phi = 3\theta - 1.107$

$\sin\phi = 1$

$\phi = \pi/2$

$3\theta - 1.107 = \pi/2$

$\theta = 0.893$ (3sf)

$0 \leq \theta < \pi$

$0 \leq 3\theta < 3\pi$

$-1.107 \leq \phi < 3\pi - 1.107$

9ci.

min value of $|\sin(3\theta - 1.107)| = 0$

$H(\theta) = 4 + 5(2\sqrt{5} \times 0)^2$

$= 4$

9cii.

$\sin(3\theta - 1.107) = 0$

$\sin\phi = 0$

$\phi = 0, \pi, 2\pi$

max value when $\phi = 2\pi$

$3\theta - 1.107 = 2\pi$

$\theta = \dfrac{2\pi + 1.107}{3}$

$= 2.46$ (3sf)

$0 \leq \theta < \pi$

$-1.107 \leq \phi < 3\pi - 1.07$